

Course in Electromechanics

Year 1

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ELECTROMECHANICS COURSE NOTES

These notes were prepared for a **Year 1 course module** in the Department of Engineering and Design at the University of Sussex. The module runs for five weeks, and absorbs 25% of student time.

The aim of the course is to develop an understanding of electromagnetic principles and their application in a range of practical electromechanical devices. Instead of the traditional emphasis on mathematical methods, this course makes extensive use of field plots generated with MagNet to explain the principles. Ampère's circuital law is a key concept; it gives a useful first estimate of the airgap flux density in devices such as electromagnets, from which the mechanical force can be calculated. Professional engineering design now uses electromagnetic simulation software such as MagNet, but users must be aware of the underlying principles and have some independent means of checking the magnitudes of quantities such as mechanical force.

In the laboratory component of the course, students have a 3-hour session using MagNet to explore the C-core electromagnet. This is an extended version of the tutorial from the Infolytica document *An Introduction to MagNet for Static 2D Modeling*, where students compare the MagNet results for airgap flux density and force with the values calculated by elementary methods. Provided the core is unsaturated, the elementary methods give remarkably good predictions of the performance. However, as the tutorial demonstrates, saturation has a dramatic effect at high current levels, and a design change is required if the predicted force is to be realized.

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1 INTRODUCTION

Electromechanical devices use electromagnetic forces to produce mechanical effects. They are very widely used in engineering; common examples are electric motors, electromagnets, and loudspeakers.

In spite of their wide diversity, these devices all depend on a small number of basic principles. The course will review these principles and show how they are exploited in practical devices, with two basic aims:

- to give an introduction to the subject that will enable application engineers to use electromagnetic devices effectively,
- to provide a foundation for more specialised courses, including the design of electromechanical devices.

Course components

The course has three closely linked components: lectures, problem workshops, and laboratories. These notes effectively form the textbook for the course, since there is no suitable book in print. Consequently, some topics are treated in greater depth than is required for completing the weekly problem worksheets or for answering examination questions. Lectures will explore the concepts and methods required for solving the worksheet problems, and illustrate the topics with practical demonstrations. Laboratories will give practical experience of two important electromechanical devices, and introduce MagNet: a leading commercial software package for electromagnetic simulation.

Course assessment

The course is assessed as follows:

June examination	70%
Mid-term test	7.5%
Workshops	7.5%
Laboratories	15%

Magnetic field plots

A magnetic field plot is often a useful way of picturing the operation of an electromagnetic device. A large number of plots have been specially prepared for these notes, using the MagNet software package from Infolytica Corporation.

References

References to books are listed in section 11, and cited in the text of the notes with the reference number in square brackets.

2 MAGNETIC EFFECTS

2.1 Conductor in a magnetic field

Magnets have the remarkable property of affecting other objects without physical contact. These effects are described in terms of the *magnetic field* of the magnet, measured by the *magnetic flux density* \mathbf{B} . This is a vector quantity, which has both magnitude and direction.

Suppose that the magnetic field is uniform, so that \mathbf{B} has a constant magnitude and direction. Consider a straight conductor of length l placed in this field, in a direction perpendicular to \mathbf{B} , as shown in figure 2-1.

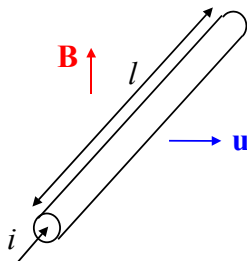


Figure 2-1: Conductor in a magnetic field.

Two effects may be observed:

- If the conductor moves with a velocity \mathbf{u} perpendicular to both the conductor and the field, there will be a voltage e generated in the conductor, given by:

$$e = Blu \quad [\text{V}] \quad (2-1)$$

- If the conductor carries a current i , there will be a force \mathbf{f} exerted on the conductor, perpendicular to both the conductor and the field. The magnitude is given by:

$$f = Bli \quad [\text{N}] \quad (2-2)$$

With the direction of i shown in figure 2-1, the direction of \mathbf{f} is the same as the direction of \mathbf{u} .

Both of these effects have wide applications in engineering. Moving-coil devices such as loudspeakers and microphones (section 7) use these effects in a direct way. Motors and generators (sections 8 and 10) are also based on these effects, although most of the force is usually exerted on steel parts rather than the conductors themselves.

2.2 Lorentz equation

Equations 2-1 and 2-2 are derived from one of the most fundamental equations of electromagnetism:

the Lorentz equation for the force \mathbf{f} on a moving charge q :

$$\mathbf{f} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \quad [\text{N}] \quad (2-3)$$

where q is the charge in coulombs, \mathbf{E} is the *electric field strength* in volts/metre, \mathbf{u} is the velocity in metres/second, and \mathbf{B} is the *magnetic flux density* in tesla (T).

The force on a charge thus has an electric component

$$\mathbf{f}_e = q\mathbf{E} \quad (2-4)$$

and a magnetic component

$$\mathbf{f}_m = q\mathbf{u} \times \mathbf{B} \quad (2-5)$$

The direction of the magnetic component is given by the right-hand screw rule of the vector product. The force \mathbf{f}_m is perpendicular to the plane containing \mathbf{u} and \mathbf{B} , with a clockwise rotation from \mathbf{u} to \mathbf{B} when viewed in the direction of \mathbf{f}_m .

Both force components can be seen in the deflection of the electron beam in a cathode-ray tube (CRT) [1]. CRTs in oscilloscopes use electrostatic deflection, exploiting \mathbf{f}_e . CRTs in video monitors and TV sets use magnetic deflection, exploiting \mathbf{f}_m .

Force on a conductor

An electric current flowing in a wire represents charge in motion, so the moving charges experience a $\mathbf{u} \times \mathbf{B}$ force when the wire is placed in a magnetic field. Consider a small length of wire $d\mathbf{l}$, carrying a current i , as shown in figure 2-2.

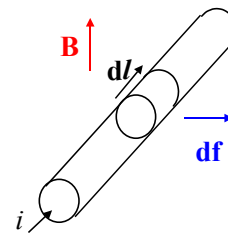


Figure 2-2: Conductor in a magnetic field

If dq is the charge in the length $d\mathbf{l}$, and \mathbf{u} is the velocity, the force acting on the current element is:

$$d\mathbf{f} = dq \mathbf{u} \times \mathbf{B} \quad (2-6)$$

The charge will travel a distance $d\mathbf{l}$ in a time dt , so the velocity is given by:

$$\mathbf{u} = \frac{d\mathbf{l}}{dt} \quad (2-7)$$

Since current is the rate of flow of charge, we have:

$$i = \frac{dq}{dt} \text{ or } dq = i dt \quad (2-8)$$

Substituting for \mathbf{u} and dq in equation 2-6 gives:

$$d\mathbf{f} = i d\mathbf{l} \times \mathbf{B} \quad (2-9)$$

The force on a conductor of length l may be found by integration. If the field is uniform and perpendicular to the conductor, then the magnitude of the force is given by equation 2-2:

$$f = Bli \quad [2-2]$$

and the direction is given by equation 2-9.

Voltage induced in a conductor

Consider the conductor shown in figure 2-1. Any free charge in the conductor will experience a force $q\mathbf{u} \times \mathbf{B}$, and will therefore start to move. Negative electrons will move towards one end, leaving behind a surplus of positive charge. This separation of charge produces an electric field \mathbf{E} inside the conductor, with a force $q\mathbf{E}$ on the charge, and the process continues until there is equilibrium:

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} \quad (2-10)$$

This is illustrated in figure 2-3.

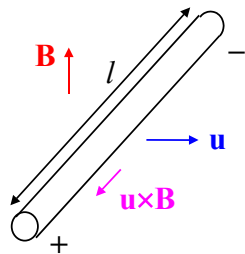


Figure 2-3: Charge separation.

If the flux density \mathbf{B} is uniform and perpendicular to the conductor, the voltage between the ends of the conductor is given by:

$$e = \int \mathbf{E} \cdot d\mathbf{l} = \int uB dl = uB \int dl = Blu \quad (2-11)$$

This is the formal derivation of equation 2-1.

2.3 Magnetic forces on iron

A different kind of magnetic effect is seen in magnets for exerting forces on objects made of iron or steel. Figure 2-4 shows an electromagnet with two coils on a U-shaped steel core attracting a steel object or *armature*. In most applications, as in this case, there is a gap between each of the magnet poles and the armature. Gaps of this kind between magnetic components are termed *airgaps*. Currents flowing in the magnet coils set up a magnetic field in the surrounding space. This field is most intense in the airgaps, resulting in a large force of attraction.

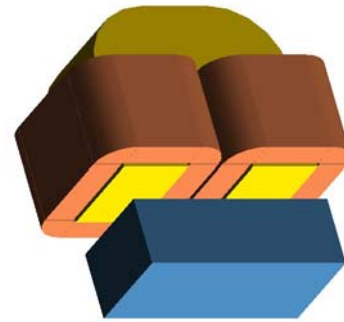


Figure 2-4: U-core electromagnet.

Figure 2-5 shows a computer-generated magnetic field plot, or *flux plot*, for the electromagnet, where the lines give a visual representation of the invisible magnetic field. The direction of the lines gives the direction of the field, and the spacing is an indication of the magnitude: the closer the lines, the greater the magnitude.

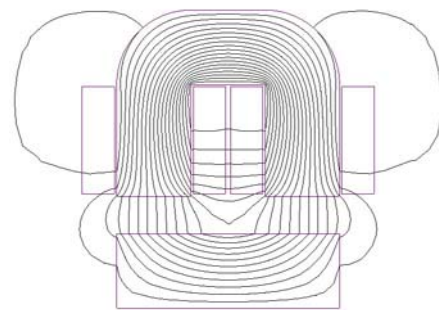


Figure 2-5: Flux plot for U-core electromagnet.

In each airgap, the magnetic field is almost uniform and at right angles to the surface of the armature.

When the magnetic field is uniform and at right angles to an iron surface, the force of attraction per unit area is given by:

$$\frac{f_m}{A} = \frac{1}{2} \frac{B^2}{\mu_0} \quad [\text{N/m}^2] \quad (2-12)$$

where f_m is the magnetic force and A is the area perpendicular to \mathbf{B} . The quantity μ_0 is the *primary magnetic constant*, with a value of $4\pi \times 10^{-7}$ H/m. A formal derivation of this equation is given in section 9, and a method of calculating the value of B is given in section 4.5.

Equation 2-12 can be applied to the electromagnet as follows. If B_g is the flux density in the airgap, f_p is the force of attraction on one pole, and A_p is the cross-sectional area of the pole, then equation 2-12 gives:

$$\frac{f_p}{A_p} = \frac{1}{2} \frac{B_g^2}{\mu_0} \quad (2-13)$$

The total force of attraction is thus:

$$f_t = 2f_p = \frac{B_g^2 A_p}{\mu_0} \quad (2-14)$$

Magnetic forces can be very large. It is possible to design an electromagnet with a flux density of 1.0 T in the airgap, which gives the following force per unit area:

$$\frac{f_m}{A} = \frac{1}{2} \frac{B^2}{\mu_0} = \frac{(1.0)^2}{2 \times 4\pi \times 10^{-7}} \approx 4 \times 10^5 \text{ N/m}^2 \quad (2-15)$$

This represents a force density of about four times the normal atmospheric pressure.

Fringing and leakage

Notice two features of the magnetic field in this electromagnet. First, the magnetic field in the airgap between the poles and the armature is not confined to the pole region, but spreads into the surrounding air; this is termed *fringing*. Secondly, some flux takes a short cut across the space between the poles, instead of crossing the airgap and passing through the armature; this is termed *leakage*.

Applications

In addition to the obvious applications in lifting magnets and devices like relays, electromagnets with electronic control can be used to eliminate mechanical contact in moving systems [2].

Figure 2-5 shows a magnetic bearing using this principle, and figure 2-6 shows the German Transrapid advanced transport system where magnetic suspension takes the place of wheels.

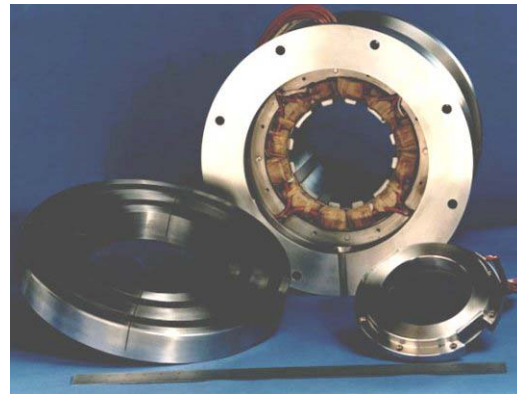


Figure 2-6: Magnetic bearing.
(Federal-Mogul Magnetic Bearings)



Figure 2-7: Advanced transport system with magnetic suspension. (Transrapid International)

2.4 Permanent magnets

Figure 2-8 shows a permanent magnet attracting a steel plate, and figure 2-9 shows the corresponding flux plot. In the electromagnet of figure 2-3, the source of the magnetic field is the current in the coils wound on the poles. In the permanent magnet of figure 2-8, the source of the magnetic field is a block of permanent-magnet material.



Figure 2-8: Permanent magnet.

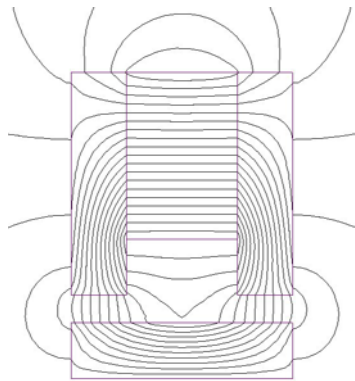


Figure 2-9: Flux plot for permanent magnet.

In many applications, permanent magnets incorporate steel poles, as in figure 2-8, because the performance can be improved significantly. However, before this can be explained, it is necessary to consider the concept of magnetic flux.

Magnetic flux

A basic concept in electromagnetism is the *magnetic flux*. If the magnetic flux density \mathbf{B} is uniform and perpendicular to a flat surface of area A , then the magnetic flux ϕ is defined to be:

$$\phi = BA \quad (2-16)$$

Thus, the flux density magnitude B is the flux per unit area. In general, if \mathbf{B} is not uniform and perpendicular to the surface, the flux is given by a surface integral:

$$\phi = \int \mathbf{B} \cdot d\mathbf{A} \quad (2-17)$$

but the simpler form of equation 2-16 will be adequate for this course.

In a flux plot such as figure 2-9, the curved lines are actually the contours of a flux function that represents flux per unit depth. If $\Delta\phi$ is the change in flux between two successive lines, and Δs is the perpendicular distance between the lines, then from equation 2-16 the flux density is:

$$B \approx \frac{\Delta\phi}{\Delta A} = \frac{\Delta\phi}{\Delta s} \quad (2-18)$$

There are equal increments of flux $\Delta\phi$ between successive pairs of lines, so widely spaced lines indicate a low flux density, and tightly packed lines indicate a high flux density.

Flux law

Associated with the concept of flux is the *flux law*, which may be stated as follows:

- For any closed region in the magnetic field, the total flux entering the region is equal to the total flux leaving.

This is analogous to Kirchoff's current law in electric circuit theory, or to the behaviour of an incompressible fluid, where the total flow into a region is equal to the total flow out. Although nothing is actually flowing in the magnetic field, the flow analogy is useful, and is implied by the name flux.

A useful application of the flux law is to find the relationship between the flux density in the steel and the flux density in the airgap for a device such as the magnet in figure 2-8. The airgap region is shown in more detail in figure 2-10.

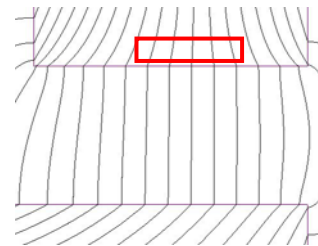


Figure 2-10: Flux plot for the airgap region.

Consider a closed region shown by the red rectangle in figure 2-10. The rectangle represents the cross-section of a box that has the same depth as the magnet poles perpendicular to the plane of the diagram. The top face of the box is in the steel pole, and the bottom face is in the airgap. Flux enters the top face of the box in the steel, and leaves the bottom face in the air. The direction of the field is nearly perpendicular to these faces of the box, so there is negligible flux out of the sides of the box. Let B_s be the value of the flux density in the steel, and B_g the value of the flux density in the airgap. Since both faces have the same area A , we have:

$$\phi = B_s A = B_g A \quad (2-19)$$

It follows that the two flux density values are equal. This is an important result, which is only valid when the field is perpendicular to the surface.

Flux concentration

In figure 2-9, there is a good deal of flux leakage between the poles and into the air behind the magnet. If the airgap is made very small, the leakage disappears, and all the flux from the permanent magnet is directed through the poles to the armature, as shown in figure 2-11.

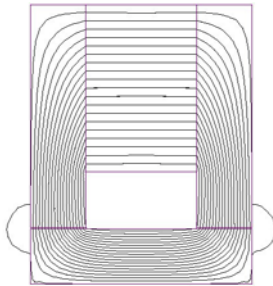


Figure 2-11: Flux plot with a small airgap.

Let A_m be the cross-sectional area of the magnet, perpendicular to the flux, as shown in figure 2-12, and A_p the corresponding pole area.

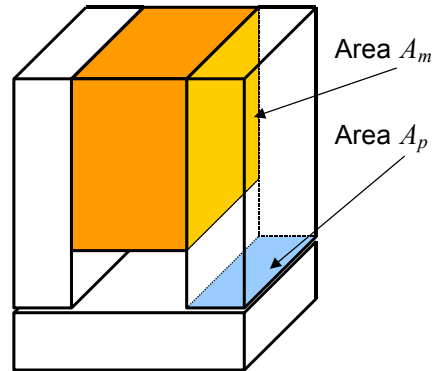


Figure 2-12: Magnet and pole areas.

The flux ϕ leaving the magnet through the area A_m enters the pole, and then leaves the pole through the area A_p . From the flux law:

$$\phi = B_m A_m = B_s A_p \quad (2-20)$$

where B_s is value of the flux density in the steel at the bottom of the pole. From equation 2-18, the flux density in the airgap is $B_g = B_s$. Thus:

$$B_g = B_s = \frac{A_m}{A_p} B_m \quad (2-21)$$

If the pole area A_p is made smaller than the magnet area A_m , equation 2-20 shows that the flux density in the airgap will be greater than the flux density in the magnet. This is the *flux concentration* effect of steel poles.

It is shown in section 4.6 that the magnet flux density B_m will remain constant under these conditions, so that reducing the area A_p will increase the airgap flux density B_g . The effect is to increase the force of attraction, for the following reason. From equation 2-14, the force is:

$$f_t = \frac{B_g^2 A_p}{\mu_0} \quad [2-14]$$

Suppose that the area A_p is halved by reducing the pole thickness. From equation 2-20, the value of B_g will be doubled, so the value of B_g^2 will be quadrupled. Therefore, the total force will double.

3 THE MAGNETIC FIELD

This method of increasing the force is used in the magnetic door catch shown in figure 2-13, which has narrow poles to concentrate the flux from an inexpensive ceramic ferrite permanent magnet.

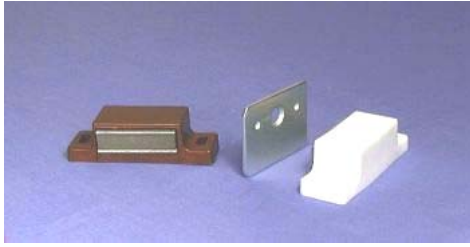


Figure 2-13: Magnetic door catch.

Properties of permanent magnets

The relative advantages of permanent magnets and electromagnets can be summarised as follows

Electromagnets	Permanent magnets
Require a current supply	Do not require a supply
Heat generated in coil	No heat generated
Field is controllable	Field is fixed*
Can be turned off	Cannot be turned off*

* The field can only be varied or the magnet turned off by changing the mechanical configuration.

Applications

Permanent magnets are widely used for providing the working flux in moving-coil devices such as loudspeakers (section 7), and in small DC and AC motors (sections 8 and 10). They are also used in magnetic holding devices and lifting magnets, in which case they may incorporate coils to counteract the effect of the permanent magnet so that the load can be released. Other applications include magnetic filtration and separation, mechanical drives and couplings, low-friction bearings, MRI analysers and scanners, and magnetrons for radar and microwave ovens. See reference [3] for further details.

3.1 Magnetic field of a conductor

Current flowing in a conductor sets up a magnetic field in the surrounding space. The magnetic flux density \mathbf{dB} that results from a current i in a small length of wire $d\mathbf{l}$ is given by a formula attributed to Ampère [1], also known as the Biot-Savart formula. The field resulting from a conductor of any shape may be determined by integration, and there is a particularly simple result for the special case of a long straight wire. The magnitude of the flux density at a distance r from the axis of the wire is given by:

$$B = \frac{\mu_0 i}{2\pi r} \quad [\text{T}] \quad (3-1)$$

where μ_0 is the *primary magnetic constant* with a value given by

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \approx 1.257 \times 10^{-6} \text{ H/m} \quad (3-2)$$

The direction of the field is tangential to a circle of radius r , so the field can be represented by circles centred on the axis as shown in the flux plot of figure 3-1. A right-hand screw rule applies to the direction of the field, which is clockwise round the circle when viewed in the direction of the current.

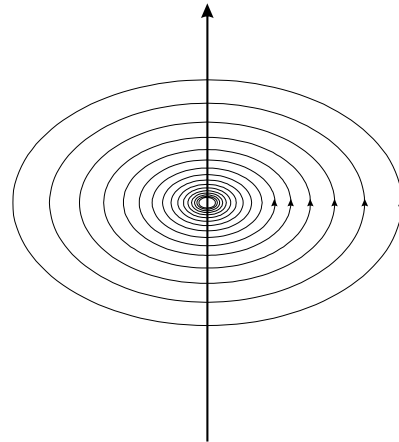


Figure 3-1: Magnetic field of a single wire.

For a wire of finite length, as shown in figure 3-2, the magnitude of the flux density at a distance r from the wire is [1]:

$$B = \frac{\mu_0 i}{4\pi r} (\cos \alpha_2 - \cos \alpha_1) \quad (3-3)$$

where α_1 and α_2 are the angles shown in figure 3-2.

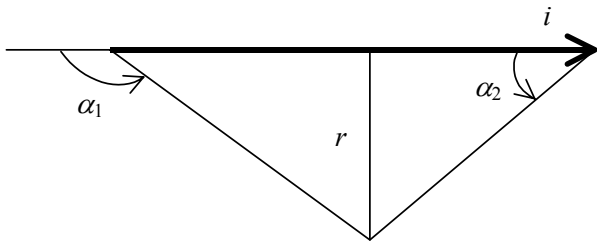


Figure 3-2: Wire of finite length.

This result can be used to calculate the field of currents flowing in wire loops of various shapes, as the following example shows.

Worked example

A wire of length 1.2 m is bent into the shape of a regular hexagon, so the length of each side is 200 mm. If the wire carries a current of 1.0 A, determine the magnitude of the flux density at the centre of the hexagon.

Solution

If lines are drawn from the two ends of one side to the centre of the hexagon, the three lines form an equilateral triangle. The angles are therefore $\alpha_1 = 120^\circ$, $\alpha_2 = 60^\circ$. The distance from the centre to the mid-point of the side is:

$$r = 100 \tan 60^\circ = 100 \times 1.732 = 173 \text{ mm}$$

The contribution of one side to the flux density is:

$$\begin{aligned} B_s &= \frac{\mu_0 i}{4\pi r} (\cos \alpha_2 - \cos \alpha_1) \\ &= \frac{4\pi \times 10^{-7} \times (\cos 60^\circ - \cos 120^\circ)}{4\pi \times 173 \times 10^{-3}} \\ &= 5.78 \times 10^{-7} \text{ T} \end{aligned}$$

The total flux density is therefore:

$$B_t = 6B_s = 6 \times 5.78 \times 10^{-7} \text{ T} = 3.47 \mu\text{T}$$

3.2 Forces on parallel currents

Consider two long parallel current-carrying conductors as shown in figure 3-3. The first conductor creates a magnetic field in the space around it. At the axis of the second conductor, this field is represented by the flux density vector \mathbf{B}_1 . Similarly, the second conductor creates a flux density \mathbf{B}_2 at the axis of the first conductor. The resulting forces will be \mathbf{f}_1 and \mathbf{f}_2 as shown in figure 3-3.

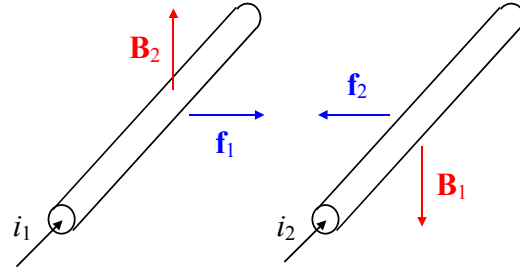


Figure 3-3: Parallel currents.

From equation 2-2, the magnitude of the force on a length l of the first conductor is:

$$f_1 = B_2 l i_1 \quad (3-4)$$

and the magnitude of the force on a length l of the second conductor is:

$$f_2 = B_1 l i_2 \quad (3-5)$$

Substituting for B in terms of the current from equation 3-1 gives the result:

$$f_1 = f_2 = \frac{\mu_0 l i_1 i_2}{2\pi r} \quad (3-6)$$

3.3 Magnetic field of parallel currents

To calculate the force on parallel conductors, we have used partial fields: only the field due to the first conductor has been used to calculate the force on the second, and *vice versa*. The total magnetic field of two currents is given by the vector sum of the fields that each would produce on its own. This is an instance of the *principle of superposition*, which is often used in electromagnetism. Note, however, that it is only valid for a linear system. It will be shown in section 4 that the principle is not valid when magnetic materials are present.

Figure 3-4 shows the flux plot when the conductors carry equal currents in the same direction. Figure 3-5 shows the corresponding plot for currents in opposite directions.

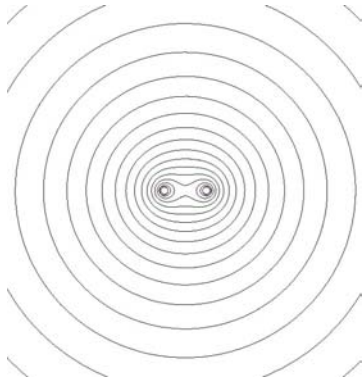


Figure 3-4: Currents in the same direction.

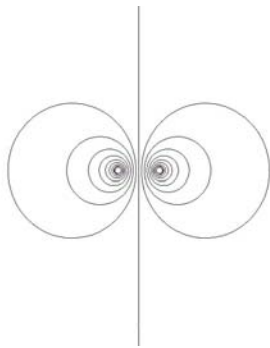


Figure 3-5: Currents in opposite directions.

With currents in the same direction, the flux lines become more like circles with increasing distance from the conductors. When the distance is large compared with the conductor separation, the field is similar to that of a single conductor carrying twice the current. In between the conductors, the fields tend to cancel, and there is a null point mid-way between the conductors where the flux density is zero.

When the currents are in opposite directions, the resultant field is completely different. Between the conductors, the fields combine. Away from the conductors, the fields tend to cancel, as can be shown by adding the fields vectorially. If d is the distance between the axes of the conductors, and r is the distance from a point mid-way between the conductors, the magnitude of the flux density is given by

$$B \approx \frac{\mu_0 i d}{2\pi r^2} \quad (3-7)$$

provided that $r \gg d$.

Equation 3-7 has an important consequence for magnetic fields in the environment. Power cables usually contain pairs of conductors carrying equal and opposite currents, so the flux density decays rapidly with distance. Its magnitude can be reduced by making the separation d very small. The external magnetic field of a normal domestic or industrial power cable is negligibly small.

The field of a single conductor, given by equation 3-1, decays as $1/r$ instead of $1/r^2$, and there is no factor corresponding to d . The external magnetic field is therefore much greater. This property of a single conductor is exploited by *induction loops* for hearing aids. An induction loop is a long wire taken round the perimeter of a room, supplied with current from a power amplifier in the public address system. The alternating magnetic field set up by the loop induces a small voltage in a special coil in the hearing aid, which can be amplified in the same way as the signal from a microphone.

3.4 Ampère's circuital law

Magnetic intensity H

When magnetic materials are considered (see section 3-8), it is necessary to introduce a new magnetic quantity H to describe the properties of the materials. In the absence of magnetic materials, H is defined as follows:

$$B = \mu_0 H \quad (3-8)$$

where H is the *magnetic intensity*, with units of amperes/metre (A/m). In terms of H , equation 3-1 for the field of a long straight wire becomes:

$$H = \frac{B}{\mu_0} = \frac{i}{2\pi r} \quad [\text{A/m}] \quad (3-9)$$

Equation 3-9 can be re-arranged to express i in terms of H :

$$i = H \cdot 2\pi r = Hl \quad [\text{A}] \quad (3-10)$$

where $l = 2\pi r$ is the circumference of a circle of radius r . Thus the current magnitude i is equal to H times the length of the path encircling the current.

Ampère's circuital law

Equation 3-10 is a particular case of a general law that applies to any closed path in the magnetic field. Figure 3-6 shows a closed path or *contour* of arbitrary shape encircling a conductor carrying a current i .

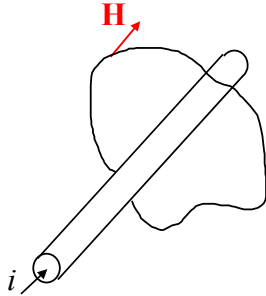


Figure 3-6: Closed path encircling a current.

In general, the magnetic intensity H will vary from point to point along the contour, and it will not be parallel to the path at all points. Consider a short length $d\mathbf{l}$ of the path, and take the scalar product of \mathbf{H} and $d\mathbf{l}$:

$$\mathbf{H} \cdot d\mathbf{l} = H dl \cos \theta \quad (3-11)$$

where θ is the angle between \mathbf{H} and $d\mathbf{l}$. The quantity $H \cos \theta$ is the component of \mathbf{H} parallel to $d\mathbf{l}$, so equation 3-11 represents the component multiplied by the length dl . If the complete contour is divided up into small pieces, an expression like equation 3-11 applies to each piece. The sum of these terms may be written as an integral, and *Ampère's circuital law* states that this integral is equal to the current in the conductor:

$$\oint \mathbf{H} \cdot d\mathbf{l} = i \quad (3-12)$$

If there are several conductors, each carrying current, passing through the contour, then the general form of equation 3-12 is:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i \quad (3-13)$$

where the summation on the right-hand side represents the algebraic sum of all the currents passing through the contour. It does not matter where the currents are placed in relation to the contour. Equation 3-13 can be used to calculate the unknown field H in many situations, as will be shown in sections 3.5 and 3.6, and it forms the basis of the magnetic circuit method introduced in section 4.

In most applications of equation 3-13, the contour is chosen to make the integration as simple as possible. If the problem has any kind of symmetry, this can be used to simplify the integration.

3.5 Field of cylindrical conductors

Field of a long straight wire

We already know that the field of a long straight wire has circular symmetry, as shown in figure 3-1. To exploit this fact in the application of Ampère's circuital law, we choose a circular contour as shown in figure 3-7.

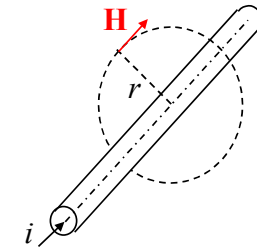


Figure 3-7: Contour for a long straight wire.

By symmetry, the direction of \mathbf{H} is tangential to the circle, and its magnitude is the same at all points on the circle. Equation 3-12 becomes:

$$\begin{aligned} i &= \oint \mathbf{H} \cdot d\mathbf{l} \\ &= \oint H dl \cos \theta = \oint H dl \\ &= H \oint dl = H \cdot 2\pi r \end{aligned} \quad (3-14)$$

Thus, the value of H is given by

$$H = \frac{i}{2\pi r} \quad [\text{A/m}] \quad (3-15)$$

This is the same expression as in equation 3-9.

Field of a solid cylinder

Consider a long solid cylinder of radius a , carrying a current I , as shown in figure 3-8. The problem is to find the magnitude of H at a distance r from the axis.

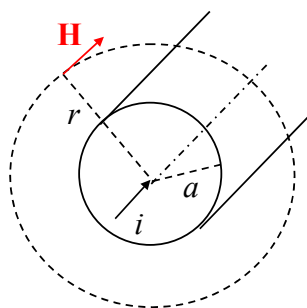


Figure 3-8: Contour for a solid cylinder.

Since this structure also has circular symmetry, we expect the field lines to be circles centred on the axis of the cylinder, but we do not yet know the magnitude of \mathbf{H} . We choose a circular contour of radius $r > a$, and follow the same argument as before:

$$i = \oint \mathbf{H} \cdot d\mathbf{l} = H \cdot 2\pi r \quad (3-16)$$

Therefore, the value of H is again given by:

$$H = \frac{i}{2\pi r} \quad [\text{A/m}] \quad (3-17)$$

Field of a hollow cylinder

Figure 3-9 shows a hollow cylinder of inner radius a_i and outer radius a_o , carrying a current i . It is required to find the magnitude of H two cases:

- outside the cylinder, at a radius $r_o > a_o$,
- inside the hole, at a radius $r_i < a_i$.

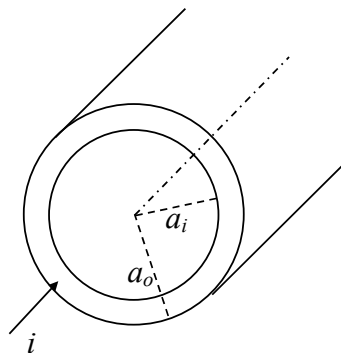


Figure 3-9: Hollow cylinder.

Figure 3-10 shows a circular contour for calculating the field H_o outside the cylinder at a radius r_o .

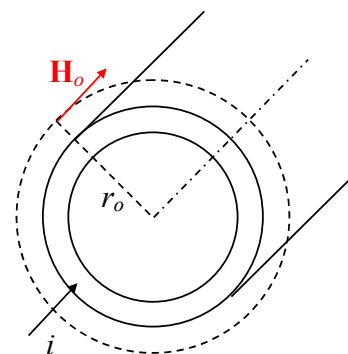


Figure 3-10: Contour for external field.

Applying Ampère's circuital law gives:

$$i = \oint \mathbf{H} \cdot d\mathbf{l} = H_o \cdot 2\pi r_o \quad (3-18)$$

$$H_o = \frac{i}{2\pi r_o} \quad [\text{A/m}] \quad (3-19)$$

Figure 3-11 shows a circular contour for calculating the field H_i inside the hole at a radius r_i .

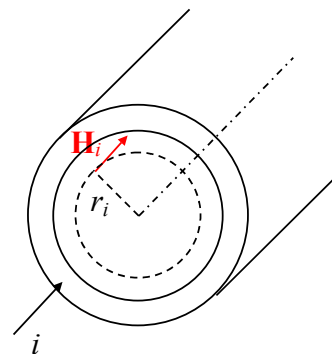


Figure 3-11: Contour for internal field.

There is no current passing through this contour, so Ampère's circuital law gives:

$$0 = \oint \mathbf{H} \cdot d\mathbf{l} = H_i \cdot 2\pi r_i \quad (3-20)$$

$$H_i = 0 \quad (3-21)$$

This interesting result, that there is no magnetic field in the hole, depends on perfect circular symmetry. If the hole is offset, there will be a field in the hole.

Field of a coaxial cable

The two conductors of a coaxial cable can be represented by a solid cylinder inside a hollow cylinder. If the conductors carry equal and opposite currents, Ampère's circuital law shows that there will be no magnetic field outside a coaxial cable provided the axes of the cylinders are coincident.

3.6 Field of a toroidal coil

Figure 3-12 shows a coil wound on a ring or toroid. The ring is the *core* of the coil, and for the present, it is assumed non-magnetic.

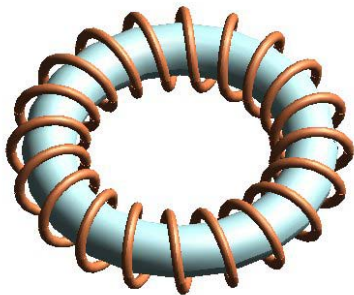


Figure 3-12: Toroidal coil.

Figure 3-13 shows the cross-section of a toroidal coil, where crosses indicate current flowing inwards and dots indicate current flowing outwards. It is required to find the magnetic field in three regions:

- inside the toroidal core, at a radius r ,
- inside the hole, at a radius r_i .
- outside the toroid, at a radius r_o ,

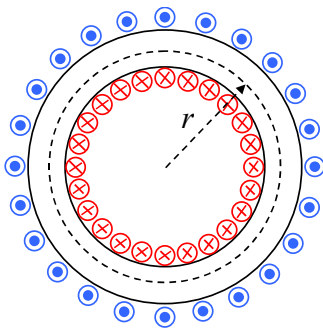


Figure 3-13: Contour for the field in the core.

Field in the core

Consider a circular contour of radius r in the core, as shown in figure 3-13. If the turns of the coil are closely spaced and uniformly distributed around the core, we may assume:

- the field direction is tangential to the circle,
- the field magnitude is constant at all points.

If the coil has N turns, and carries a current i , then the total current passing through the contour is Ni . Ampère's circuital law (equation 3-13) gives:

$$\sum i = Ni = \oint \mathbf{H} \cdot d\mathbf{l} = H \cdot 2\pi r \quad (3-22)$$

Therefore, the value of H in the core is given by

$$H = \frac{Ni}{2\pi r} \quad [\text{A/m}] \quad (3-23)$$

Note that the value of H will vary over the cross-section of the core, since r is not the same at all points, but this variation will be ignored. If $D = 2r$ is the mean diameter of the toroid, then the mean value of H is

$$H = \frac{Ni}{\pi D} \quad [\text{A/m}] \quad (3-24)$$

and the corresponding flux density is

$$B = \mu_0 H = \frac{\mu_0 Ni}{\pi D} \quad [\text{T}] \quad (3-25)$$

Field in the hole

Figure 3-14 shows the integration contour for finding the field in the hole.

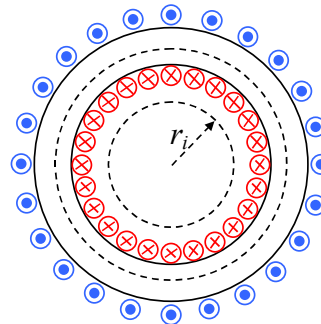


Figure 3-14: Contour for the field in the hole.

As with the hollow cylinder, there is no current passing through this contour, so we have:

$$0 = \oint \mathbf{H} \cdot d\mathbf{l} = H_i \cdot 2\pi r_i \quad (3-26)$$

$$H_i = 0 \quad (3-27)$$

Field outside the toroid

Figure 3-15 shows the integration contour for finding the field outside the toroid.

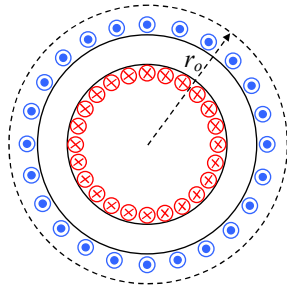


Figure 3-15: Contour for the exterior field.

Current from each turn of the coil passes through this contour twice: once on the inside of the core, and once on the outside. Since the currents are in opposite directions, they cancel out. Equation 3-13 gives:

$$\sum I = 0 = \oint \mathbf{H} \cdot d\mathbf{l} = H_o \cdot 2\pi r_o \quad (3-28)$$

Therefore, the value of H_o outside the toroid is zero.

Applications

A perfect toroidal coil has the remarkable property that the magnetic field is confined entirely to the core: there is no external magnetic field. Practical coils will depart from this ideal because the turns are not perfectly distributed, but the external field is usually very small. For this reason, toroids are widely used in transformers and inductors. Figure 3-16 shows a small toroidal inductor of this kind.



Figure 3-16: Toroidal inductor.
(RS Components Ltd)

3.7 Magnetic flux and inductance

Flux linkage

Magnetic flux was defined in equation 2-16:

$$\phi = BA \quad [2-16]$$

For the toroidal coil, B is the flux density in the core, and A is the cross-sectional area of the core. Another quantity known as the *flux linkage* is important for coils. It is defined as the sum of the fluxes for each turn of the coil. If the coil has N turns, and each turn has the same flux ϕ , then the flux linkage is

$$\lambda = N\phi = NBA \quad [\text{Wb}] \quad (3-29)$$

In many cases, the flux is not the same for each turn, so the general definition of flux linkage is:

$$\lambda = \sum \phi \quad [\text{Wb}] \quad (3-30)$$

where the summation is over the number of turns in the coil, and the individual flux values are given by equation 2-16. Equation 3-29 will be used in this course.

Self-inductance and stored energy

Suppose that a coil carries a current i . In the absence of magnetic materials, the magnetic flux density B at any point will be proportional to i . It follows that the flux and the flux linkage are also proportional to i , so we may put:

$$\lambda = N\phi = Li \quad (3-31)$$

where L is a constant known as the *self-inductance* of the coil. It will be shown in section 5 that this is consistent with the self-inductance of circuit theory. Re-arranging equation 3-31 gives:

$$L = \frac{\lambda}{i} \quad [\text{henry, H}] \quad (3-32)$$

From circuit theory, the magnetic energy stored in the coil is given by:

$$W_m = \frac{1}{2} Li^2 \quad [\text{joule, J}] \quad (3-33)$$

Substituting for Li from equation 3-31 gives:

$$W_m = \frac{1}{2} Li^2 = \frac{1}{2} Li \cdot i = \frac{1}{2} \lambda i \quad (3-34)$$

Compare this with the electric energy stored in a capacitor charged to a voltage v :

$$W_e = \frac{1}{2} C v^2 = \frac{1}{2} q v \quad (3-35)$$

where C is the capacitance and q is the charge.

Inductance of a toroidal coil

For the toroidal coil considered in section 3.6, we have:

$$\lambda = N\phi = NBA = NA \cdot \frac{\mu_0 Ni}{\pi D} \quad (3-36)$$

The self-inductance is therefore given by:

$$L = \frac{\lambda}{i} = \frac{\mu_0 N^2 A}{\pi D} \quad [\text{H}] \quad (3-37)$$

Note that L depends on the coil geometry and the square of the number of turns. This expression for the inductance is valid only for a non-magnetic core.

Units of μ_0

Re-arranging equation 3-37 gives:

$$\mu_0 = \frac{\pi DL}{N^2 A} \left[\frac{\text{m} \cdot \text{H}}{\text{m}^2} = \text{H/m} \right] \quad (3-38)$$

Thus the units of μ_0 are H/m.

3.8 Magnetic materials

Introduction

Suppose that the toroidal core is wound on a core of magnetic material such as iron or steel. Experimentally, the inductance is much larger, and it varies with the current in the coil. Thus, we may put:

$$L' = \mu_r L \quad (3-39)$$

where μ_r is a dimensionless property of the material known as the *relative permeability*. Since the inductance is still defined by equation 3-32, it follows that the flux linkage λ , and therefore the flux density B , has increased by the same factor μ_r .

We choose to retain the definition of H in terms of the current through Ampère's circuital law (equation 3-13). Since B has changed, it follows that equation 3-8 is no longer valid:

$$B \neq \mu_0 H \quad (3-40)$$

Instead, we must have:

$$B = \mu_r \mu_0 H = \mu H \quad (3-41)$$

where $\mu = \mu_r \mu_0$ is the *permeability*. From equation 3-41, the relative permeability is:

$$\mu_r = \frac{B}{\mu_0 H} \quad (3-42)$$

The effect of the magnetic material is to make the flux density B much larger than it would have been with the coil alone. Physically, this happens because the original field from the coil has magnetised the material of the core. The magnetised material sets up its own field, adding to the original field of the coil.

When considering magnetic materials it is important to preserve the distinction between \mathbf{B} and \mathbf{H} . Ideas of cause and effect can be helpful:

- \mathbf{H} is the cause, related to currents through Ampère's circuital law.
- \mathbf{B} is the effect, related to observable quantities such as force through the Lorentz equation.

Note that the unit of \mathbf{H} is the ampere/metre (A/m), whereas the unit of \mathbf{B} is the tesla (T).

Engineering materials

The relative permeability μ_r is very close to 1 except for two groups of materials:

- Ferromagnetics: iron and iron alloys (steels), cobalt, nickel. These are electrical conductors.
- Ferrimagnetics: ferrites – based on oxides of iron. These are electrical insulators.

For these groups, μ_r is very large and variable. Everything else is effectively non-magnetic for engineering purposes. Magnetic materials are further classified as *soft* and *hard*.

- Soft magnetic materials have very little residual magnetisation when the external source of \mathbf{H} is removed.
- Hard magnetic materials retain considerable magnetisation when the external source of \mathbf{H} is removed.

Hard magnetic materials are permanent magnets, which will be considered in section 4.3. Only soft magnetic materials are considered in the present section.

Measurement of B and H

The relationship between B and H can be measured with two coils wound on a ring sample of the material:

- The primary coil carries a current i , giving a known value of H in the ring (equation 3-24).
- The corresponding value of B is determined from the voltage induced in the secondary coil when the primary current changes.

From Faraday's law (see section 5, equation 5-2) the voltage is:

$$e = \frac{d\lambda}{dt} = NA \frac{dB}{dt} \quad (3-43)$$

The voltage is applied to the input of an electronic integrator, giving an output proportional to B :

$$v_o = k \int e dt = kNA \int \frac{dB}{dt} dt = kNAB \quad (3-44)$$

Relationship between B and H

Figure 3-17 shows the relationship between B and H from measurements on a ring sample of a soft magnetic material.

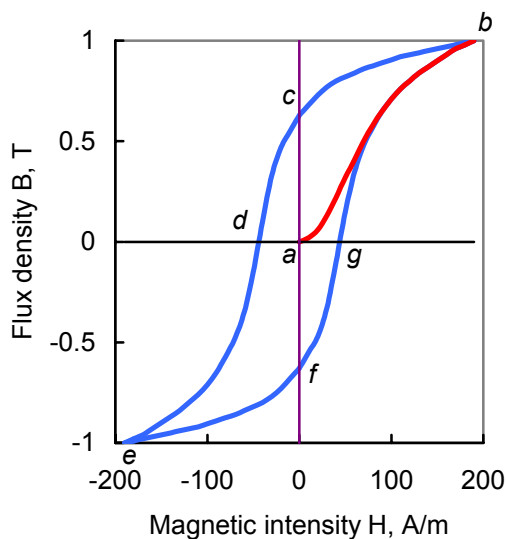


Figure 3-17: B/H relationship for soft material.

If the material is initially demagnetised, and the value of H is progressively increased by raising the current i in the primary coil, the resulting B follows a curve such as the red curve ab in figure 3-17.

When the value of H is reduced to zero, B does not fall to zero. Instead, the blue curve bc is traversed, and H must be reversed to bring the value of B to zero at point d . Further increase of negative H takes the curve to point e . If H is gradually changed from negative to positive, the curve $efgb$ is traversed, and the cycle repeats.

The curve ab is known as the *initial magnetisation curve*, and the loop $bcdefg$ is the *hysteresis loop*. It may be shown that the area of this loop represents energy lost per unit volume in taking the material round one cycle of magnetisation.

Magnetisation characteristic

For many purposes the difference between the two sides of the hysteresis loop in figure 3-17 can be ignored. An average value is reasonably representative of the material properties, and the resulting curve is known as the *magnetisation characteristic*. Figure 3-18 shows part of the magnetisation characteristic for a typical silicon steel of the kind used in transformers and motors. Further graphs are given in section 12.1.

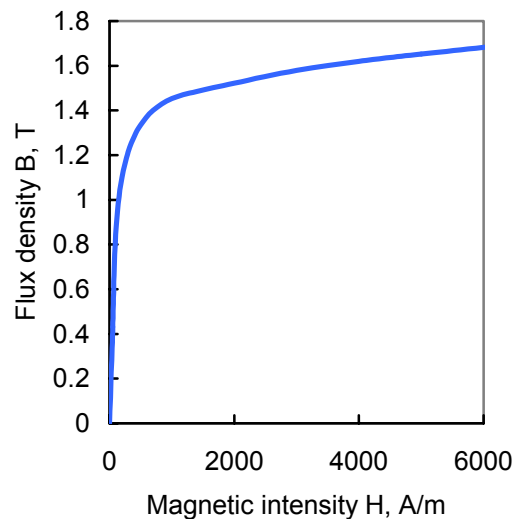


Figure 3-18: Characteristic for silicon steel.

The shape of the curve in figure 3-18 is typical of soft magnetic materials. It has two distinct regions, joined by a ‘knee’:

- A steeply rising initial part, where a small increase in H gives a large increase in B .
- A saturated region, above about 1.5 T, where any further increase in B requires a large increase in H .

Equation 3-42 gives the relative permeability μ_r in terms of B and H . Figure 3-19 shows the resulting variation of μ_r with B , calculated for the same silicon steel.

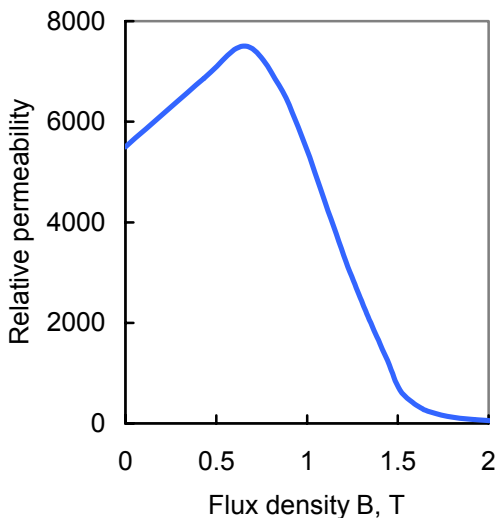


Figure 3-19: Relative permeability.

4 MAGNETIC CIRCUITS

4.1 The magnetic circuit concept

The soft magnetic materials, considered in section 3.8, have a remarkable property. If the relative permeability is high enough, they act as ‘conductors’ of magnetic flux. To demonstrate this, figure 4-1 shows a ring core, with a concentrated coil instead of the uniformly wound coil of section 3.6.

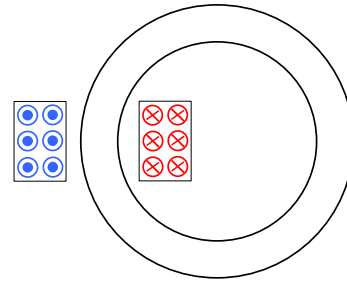


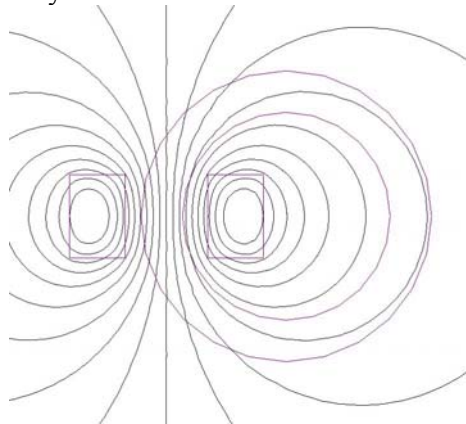
Figure 4-1: Ring core with concentrated coil.

Suppose that cores of different materials are used, with relative permeability values ranging from 1 to 1000. The resulting flux plots are shown in figure 4-2. A value of 1 means that the material is non-magnetic, so this flux plot is the same as the field of the coil without a core.

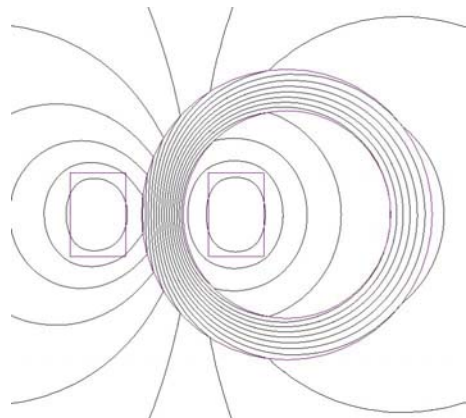
With increasing values of relative permeability, more of the coil flux is diverted from the air to the core. When $\mu_r = 1000$, the flux is virtually confined to the core, giving the same distribution of flux density as a uniformly wound toroidal coil. The exact shape and location of the coil have little effect on the flux distribution in the core. The core that guides the flux in a circular path is a simple example of a *magnetic circuit*.

From the graph of relative permeability for silicon steel in figure 3-19, the value of μ_r will exceed 1000 if the value of B is less than 1.5 T. The material is *unsaturated*, and it acts as an effective flux guide. However, if the flux density is significantly greater than 1.5 T, the material is *saturated* and the value of μ_r decreases rapidly. It then ceases to be an effective flux guide.

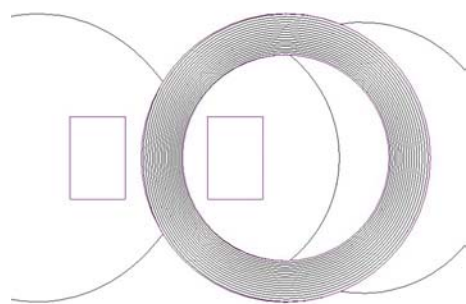
Practical magnetic circuits normally exploit the unsaturated properties of magnetic materials. Designers need to ensure that the material is not saturated. Users need to be aware of the possibility of saturation if currents exceed the design values, which may alter the behaviour of the device.



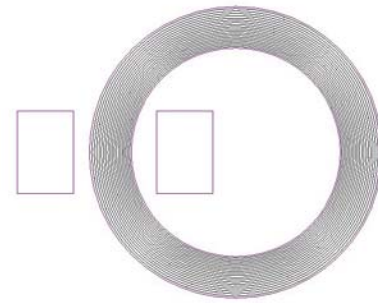
$\mu_r = 1$



$\mu_r = 10$



$\mu_r = 100$



$\mu_r = 1000$

Figure 4-2: Effect of core permeability.

Suppose that an airgap is cut in the core, as shown in figure 4-3. The resulting flux plot is shown in figure 4-4.

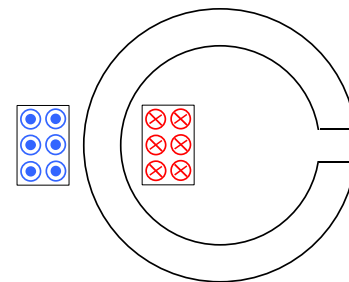


Figure 4-3: Ring core with an airgap.

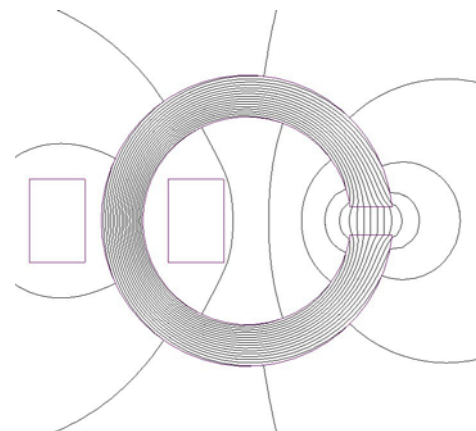


Figure 4-4: Flux plot for a ring with a airgap.

Although the relative permeability of the core is still 1000, the flux is no longer confined to the core. As with the magnets considered in section 2, there is some fringing of the field at the edges of the airgap, and some leakage of flux from one side of the ring to the other, across the air space.

Although the field is significantly different from that of a closed ring, the ring with an airgap is another example of a magnetic circuit.

The formal concept of the magnetic circuit depends on the flux law, which was introduced in section 2.4:

- For any closed region in the magnetic field, the total flux entering the region is equal to the total flux leaving.

This statement is exactly true if the flux in the air is included in addition to the flux in the core. To a first approximation, we may ignore the effects of leakage and fringing, and suppose that the flux in the core is the same for all cross-sections. With this approximation, the magnetic structure is analogous to a series electric circuit, where the current is the same at all points. It is analysed in a similar way, with Ampère's circuital law taking the place of Kirchhoff's voltage law.

The magnetic circuit concept gives a quick and simple way of getting an approximate answer. Where an accurate solution is required, powerful computer modelling methods are available. Even when computer methods are used, it is valuable to check the result by a magnetic circuit calculation.

4.2 Magnetic circuits without airgaps

Figure 4-5 shows a simple magnetic circuit without an airgap, where the dotted circular contour of length l is the mean circumference of the ring. The coil has N turns and carries a current i .

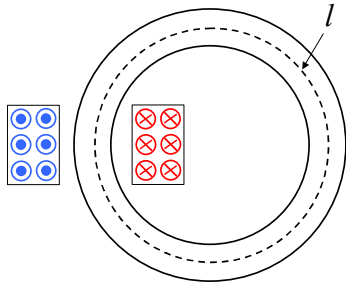


Figure 4-5: Magnetic circuit without an airgap.

The core is assumed to have a constant cross-sectional area. It follows from the flux law that the flux density B is constant, so H is also constant. Applying Ampère's circuital law gives:

$$\sum i = Ni = \oint \mathbf{H} \cdot d\mathbf{l} = Hl \quad (4-1)$$

The value of H in the ring is therefore given by

$$H = \frac{Ni}{l} \quad [\text{A/m}] \quad (4-2)$$

The corresponding value of B is determined from the curve for the material. There are two cases:

1. The coil current i is specified, and it is required to find the corresponding value of B .
2. The value of B is specified, and it is required to find the corresponding coil current i .

For case 1, the value of H is determined from equation 4-2, and the value of B found from the material curve. For case 2, the process is reversed: the value of H is found from the curve, and then substituted in equation 4-2 to get i .

Worked example 4-1

A ring of silicon steel has a mean diameter of 80 mm, and there is a coil of 1200 turns wound on the ring. Determine the current in the coil that will give a flux density of 1.1 T in the ring.

Solution

From the curve in figure 12-2, $H = 200$ A/m.

Equation 4-2 gives:

$$i = \frac{Hl}{N} = \frac{200 \times \pi \times 80 \times 10^{-3}}{1200} \text{ A} = 41.9 \text{ mA}$$

Non-linearity

An important consequence of the non-linear relationship between B and H is that B is not proportional to the current in the coil. If the calculation of worked example 4-1 is repeated for a flux density of 1.4 T instead of 1.1 T, the value of H increases from 200 to 700 A/m, and the coil current increases from 41.9 mA to 147 mA. Thus, a 27% increase in B requires a 250% increase in current. It follows that the inductance given by equation 3-32 is not constant but varies over a wide range.

A further consequence of non-linearity is that the principle of superposition no longer holds for the magnetic flux density. Suppose that the ring in worked example 4-1 is wound with two coils, each of 1200 turns. A current of 41.9 mA in either coil will give a flux density of 1.1 T in the ring, but the same current in both coils will not give a flux density of 2.2 T. The total value of H is the sum of the contributions from each coil, so H increases from 200 to 400 A/m, and the flux density increases from 1.1 to 1.3 T.

4.3 Magnetic circuits with airgaps

Figure 4-6 shows a magnetic circuit with an airgap. The effective length of the steel ring, in the direction of the magnetic flux, is the dotted path of length l_s , and the length of the airgap is l_g . The coil has N turns and carries a current i .

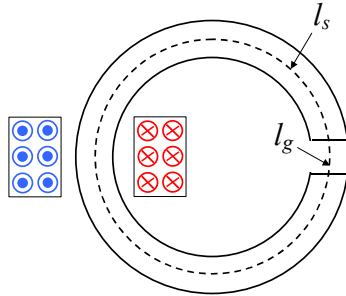


Figure 4-6: Ring core with an airgap.

The first step is to establish the relationship between the values of B and H in the airgap and the values in the steel. Let ϕ_s be the flux in the steel, and ϕ_g the flux in the airgap. From the flux law, $\phi_s \approx \phi_g$. If A_s is the cross-sectional area of the steel, and A_g is the cross-sectional area of the airgap, then:

$$B_s A_s \approx B_g A_g \quad (4-3)$$

Often, as in figure 4-6, the two areas are equal. In this case, we have the result:

$$B_s \approx B_g \quad (4-4)$$

The closeness of the approximation depends on the amount of fringing and leakage: see section 4.5. For simplicity, these effects will be ignored, and it will be assumed that $B_s = B_g$. However, the values of H cannot be the same. In the airgap, equation 3-8 gives

$$H = \frac{B}{\mu_0} \quad (4-5)$$

In the steel, H is obtained from the magnetisation curve for the given value of B , and this will be different from the value given by equation 4-5.

If the value of B is specified, then the values of H can be determined in the steel and in the airgap. The required current in the coil can then be determined from Ampère's circuital law as follows:

$$\sum i = Ni = \oint \mathbf{H} \cdot d\mathbf{l} = H_s l_s + H_g l_g \quad (4-6)$$

This gives the result

$$i = \frac{H_s l_s + H_g l_g}{N} \quad (4-7)$$

Unlike the previous problem, it is not straightforward to find the value of B that results from a given coil current i , because of the non-linear relationship between H_s and B_s .

In many practical magnetic circuits, the value of $H_g l_g$ greatly exceeds the value of $H_s l_s$, so the airgap has a dominant effect.

Worked example 4-2

A ring of silicon steel has a mean diameter of 80 mm, and there is a coil of 1200 turns wound on the ring. An airgap of length 3 mm is cut in the ring. Determine the current in the coil that will give a flux density of 1.1 T in the ring.

Solution

We have $B_s = B_g = 1.1$ T.

From the curve in figure 12-2, $H_s = 200$ A/m.

From equation 4-5, H_g is given by:

$$H_g = \frac{B_g}{\mu_0} = \frac{1.1}{4\pi \times 10^{-7}} \text{ A/m} = 875 \text{ kA/m}$$

Thus:

$$H_s l_s = 200 \times (80\pi - 3) \times 10^{-3} = 49.7 \text{ A}$$

$$H_g l_g = 875 \times 10^3 \times 3.0 \times 10^{-3} \text{ A} = 2.63 \text{ kA}$$

Equation 4-7 gives:

$$i = \frac{H_s l_s + H_g l_g}{N} = \frac{49.7 + 2.63 \times 10^3}{1200} \text{ A} = 2.23 \text{ A}$$

Non-linearity

A useful effect of the airgap in a magnetic circuit is to reduce the non-linearity. If the calculation of worked example 4-2 is repeated for a flux density of 1.4 T instead of 1.1 T, the value of H_s increases from 200 to 700 A/m. However, $H_s l_s$ is still much smaller than $H_g l_g$, and the coil current increases from 2.23 A to 2.93 A. In this case, a 27% increase in B requires a 31% increase in current. Compare this with the previous case of the ring without an airgap, where the same increase in B required a 250% increase in the current. The non-linearity is greatly reduced, and the inductance is almost constant.

Variable cross-sectional area

The air and iron parts of the magnetic circuit may have different cross-sectional areas, as shown in figure 4-7, where the air cross-sectional area A_g is larger than the steel cross-sectional area A_s .

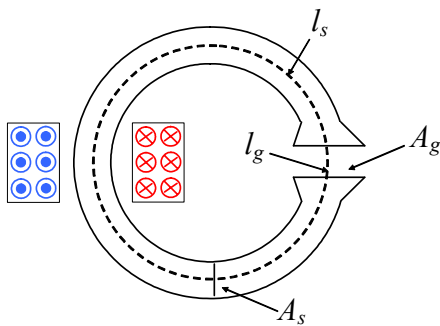


Figure 4-7: Ring core with an airgap.

If the flux values are similar, it follows from equation 4-3 that the flux density in the steel will be larger than the flux density in the airgap:

$$\frac{B_s}{B_g} \approx \frac{A_g}{A_s} \quad (4-8)$$

4.4 Electric circuit analogy

The quantity Ni is the driving force in a magnetic circuit, with a similar function to the electromotive force or voltage in an electric circuit. It is therefore termed the *magnetomotive force*, with the symbol \mathcal{F} . Corresponding to the current in an electric circuit, we have the flux ϕ for a magnetic circuit. Equation 4-6 can be re-cast in terms of flux if we express H in terms of B through equation 3-41:

$$B = \mu_r \mu_0 H = \mu H \quad [3-41]$$

This gives:

$$\begin{aligned} \mathcal{F} = Ni &= H_s l_s + H_g l_g = \frac{B_s}{\mu_s} l_s + \frac{B_g}{\mu_g} l_g \\ &= \frac{\phi}{\mu_s A_s} l_s + \frac{\phi}{\mu_g A_g} l_g \\ &= \phi \left(\frac{l_s}{\mu_s A_s} + \frac{l_g}{\mu_g A_g} \right) = \phi (R_s + R_g) \end{aligned} \quad (4-9)$$

In equation 4-9, A_s is the cross-sectional area of the steel, and A_g is the cross-sectional area of the airgap (see figure 4-7). The quantities R_s and R_g are termed the *reluctances* of the two parts of the circuit. They are analogous to resistances in an electric circuit, and are given by:

$$R_s = \frac{l_s}{\mu_s A_s}, \quad R_g = \frac{l_g}{\mu_g A_g} \quad (4-10)$$

Equation 4-9 is identical in form to the voltage equation of the electric circuit in figure 4-8:

$$v = i(R_1 + R_2) \quad (4-11)$$

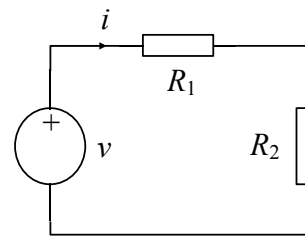


Figure 4-8: Analogous electric circuit.

Table 4-1 on the next page compares electric and magnetic circuits. The analogy is useful because the familiar ideas of electric circuits can suggest ways of handling the less familiar concepts of magnetic circuits. In problems that are more complex, where the magnetic circuit contains parallel paths, this approach can be very helpful.

Table 4-1: Circuit analogues.

Magnetic circuit	Electric circuit
$F = \phi(R_s + R_g)$	$v = i(R_1 + R_2)$
$f = \text{MMF [A]}$	$v = \text{EMF [V]}$
$\phi = \text{flux [Wb]}$	$i = \text{current [A]}$
$\mathfrak{R} = \text{reluctance [A/Wb]}$	$R = \text{resistance } [\Omega]$
$R = \frac{l}{\mu_r \mu_0 A} = \frac{l}{\mu A}$	$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$
Flux law	Kirchhoff's current law
Ampère's circuital law	Kirchhoff's voltage law

4.5 Magnetic circuit problems

When there are airgaps in the magnetic circuit, the assumption that the flux is constant around the circuit is an oversimplification. Consider the ring core of worked example 4-2. The ring has a constant cross-sectional area, so the flux density should be constant if the flux is constant. Figure 4-9 shows a computer-generated flux plot together with a shaded plot of the flux density magnitude, where values are represented by colours ranging from blue for minimum to red for maximum. In this case, the fringing and leakage fluxes have a significant effect, and the flux density is far from constant.

The computer results show that the flux density varies from 1.05 T in the middle of the airgap to 1.65 T on the opposite side of the core. In the worked example, a flux density of 1.1 T was assumed for all parts of the core and the airgap. The simple magnetic circuit method has therefore under-estimated the value of H in most of the core.

In this case, the error has had little effect on the result. With a current of 2.23 A, the true value of the airgap flux density is 1.05 T instead of 1.1 T. However, if a higher airgap flux density were required, the error would be much greater. With a coil current of 2.93 A, the magnetic circuit method gives an airgap flux density of 1.4 T, but the true value is only 1.11 T. The flux density on the

opposite side of the core is then 1.77 T, which is well into the saturation region of the magnetisation characteristic for the material. To prevent saturation, it would be necessary to increase the cross-sectional area of the core in this region.

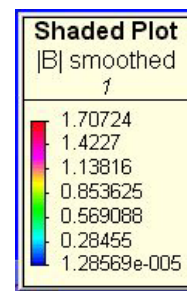
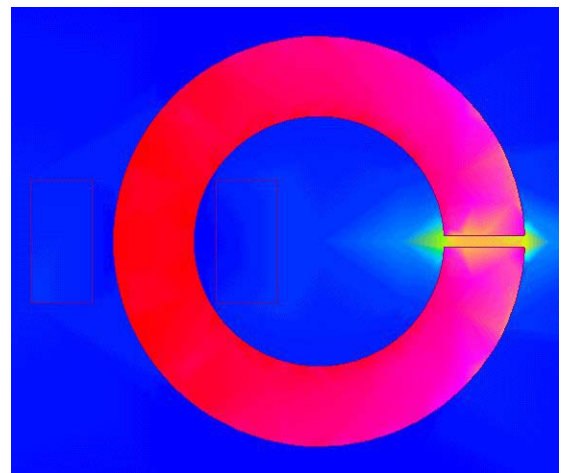
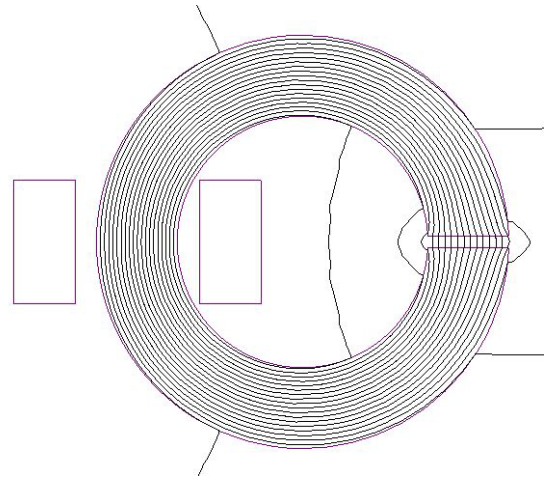


Figure 4-9: Flux plot and shaded plot of $|B|$.

4.6 Electromagnets

The magnetic circuit concept can be used to calculate the flux density in electromagnets of the kind considered in section 2.3. Figure 4-10 shows the cross-section of a simple electromagnet. Coils are wound on the steel poles, which are linked by a steel yoke to form the magnet core.

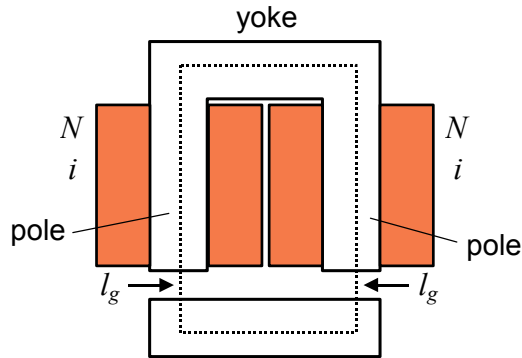


Figure 4-10: Simple electromagnet.

Applying Ampère's circuital law to the dotted contour in figure 4-10 gives:

$$\sum i = 2Ni = \oint \mathbf{H} \cdot d\mathbf{l} = H_s l_s + 2H_g l_g \quad (4-12)$$

where H_s is the value of H in the steel parts, which is assumed to be constant, and l_s is the total length of the steel path. At the boundary between the steel pole and the airgap, we have from equation 4-4:

$$B_s = B_g \quad [4-4]$$

Equation 4-4 can be expressed in terms of H :

$$\mu_r \mu_0 H_s = \mu_0 H_g \quad (4-13)$$

Thus:

$$H_s = \frac{H_g}{\mu_r} \quad (4-14)$$

If the steel is unsaturated, so that μ_r is very large, equation 4-14 shows that H_s will be very much smaller than H_g . Provided the airgap length is not too small, it follows that:

$$H_s l_s \ll 2H_g l_g \quad (4-15)$$

Equation 4-12 therefore becomes:

$$2Ni \approx 2H_g l_g \quad (4-16)$$

The flux density in the airgap is thus:

$$B_g = \mu_0 H_g = \frac{\mu_0 Ni}{l_g} \quad (4-17)$$

The approximation in equation 4-16 is equivalent to neglecting the reluctance of the steel parts in comparison with the reluctance of the airgap, as the following consideration shows. From equation 4-9, with $2N$ in place of N , we have:

$$F = 2Ni = \phi(R_s + R_g) \quad (4-18)$$

The reluctances are:

$$R_s = \frac{l_s}{\mu_r \mu_0 A_s} \quad (4-19)$$

$$R_g = \frac{2l_g}{\mu_r \mu_0 A_g} \quad (4-20)$$

If the steel relative permeability μ_r is large, then $R_s \ll R_g$, so equation 4-18 becomes:

$$2Ni \approx \phi R_g = B_g A_g \cdot \frac{2l_g}{\mu_0 A_g} = \frac{2B_g l_g}{\mu_0} \quad (4-21)$$

This gives the same result for B_g as equation 4-17.

Since the steel parts no longer feature in the equation for B_g , it follows that the exact distribution of H – and therefore B – in the steel is unimportant. In fact, there will be considerable variation, as the flux plot in figure 4-10 shows, and the designer may need to increase the thickness of part of the core to prevent saturation.

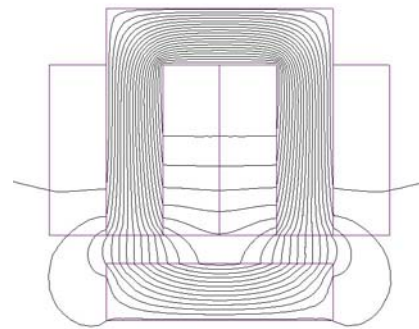


Figure 4-11: Flux plot for simple electromagnet.

The total force is given by equation 2-14:

$$f_t = \frac{B_g^2 A_p}{\mu_0} \quad [2-14]$$

If the flux density is calculated from equation 4-17, the error in the force given by equation 2-14 is generally less than 10% in simple electromagnets if the steel is unsaturated.

Accurate calculation of the airgap flux density and the force requires electromagnetic simulation software, particularly if parts of the magnetic circuit are saturated. Nevertheless, equations 4-17 and 2-14 are useful preliminary design tools, and they give a valuable check on the results of computer simulation.

Alternative coil position

Although it is common to put coils on the two poles, as in figure 4-10, an alternative is a single coil on the yoke, as shown in figure 4-12. Because of the flux-guiding properties of the steel core, the position makes very little difference to the magnet performance. The flux plot in figure 4-13 is very similar to the one in figure 4-11, and the force of attraction is virtually unchanged.

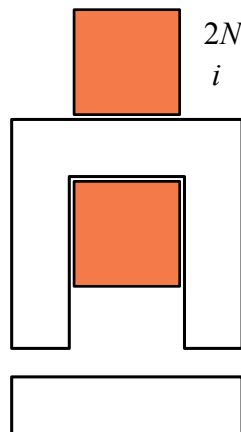


Figure 4-12: Electromagnet with a single coil.

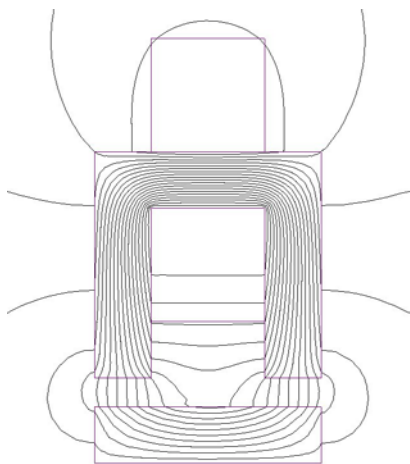


Figure 4-13: Flux plot for electromagnet.

4.7 Permanent magnets

Ampère's circuital law applies to permanent magnets as well as electromagnets. Consider the dotted contour in figure 4-14.

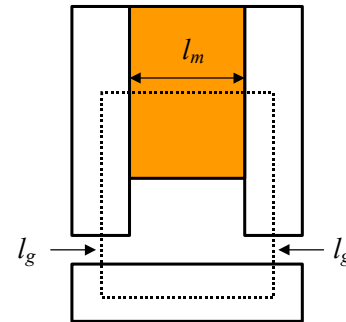


Figure 4-14: Contour for Ampère's circuital law.

There is no current passing through this contour, so we have:

$$0 = \oint \mathbf{H} \cdot d\mathbf{l} = H_m l_m + H_s l_s + 2H_g l_g \quad (4-22)$$

As with the electromagnet, H_s is the value of H in the steel parts, which is assumed to be constant, and l_s is the total length of the steel path. By a similar argument, if the steel poles are unsaturated, $H_s l_s$ will be small in comparison with the other terms in equation 4-22, so we have:

$$H_m l_m \approx -2H_g l_g \quad (4-23)$$

If the contour is drawn in the direction of the magnetic flux, H_g is positive. Equation 4-23 shows that H_m must be negative, which is in the opposite direction to B_m in the permanent magnet.

In the airgap we have $B_g = \mu_0 H_g$. If leakage is neglected, we also have the relationship between B_m and B_g from the flux law:

$$B_g = \frac{A_m}{A_p} B_m \quad [2-19]$$

With these relationships, equation 4-23 can be recast in terms of H_m and B_m :

$$H_m \approx -2 \frac{B_m}{\mu_0} \cdot \frac{A_m}{A_p} \cdot \frac{l_g}{l_m} \quad (4-24)$$

Equation 4-24 is the equation of a straight line through the origin. No matter what may be the relationship between B and H for the permanent-magnet material, the magnetic circuit imposes the condition that the working values H_m and B_m must lie on this line.

Permanent-magnet materials

Figure 4-15 shows the hysteresis loop for a permanent-magnet material.

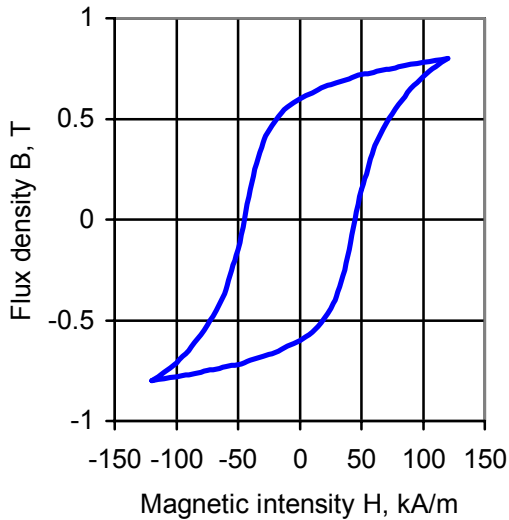


Figure 4-15: Material B/H characteristic.

It follows from equation 4-24 that the working part of the material curve for a permanent magnet is the second quadrant, where H is positive and B is negative, as shown in figure 4-16. Because H is acting in a direction to demagnetise the material, this is known as the *demagnetisation characteristic*.

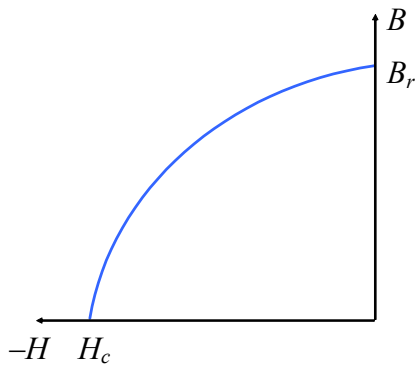


Figure 4-16: Demagnetisation characteristic.

Two important properties of a permanent magnet material are the points where the curve intersects the axes.

- The *remanence* B_r is the value of B that remains in the material when H is reduced to zero.
- The *coercivity* H_c is the value of H that must be applied to the material to bring B to zero.

Typical values of B_r and H_c are given in table 4-1 for some representative permanent-magnet materials [3]. The higher the value of H_c , the more difficult it is to demagnetise the material.

Table 4-1: Permanent-magnet materials.

Material	B_r (T)	$-H_c$ (kA/m)
1% carbon steel	0.9	4
35% cobalt steel	0.9	20
Alnico 4	1.0	120
Ceramic ferrite	0.4	290
Samarium cobalt	1.0	480
Neodymium iron boron	1.2	910

Magnet working point

The working values of B and H in the permanent-magnet material is given by the intersection of the line of equation 4-24 and the material curve, as shown in figure 4-17. By analogy with electric circuits, the line is often called a *load line*.

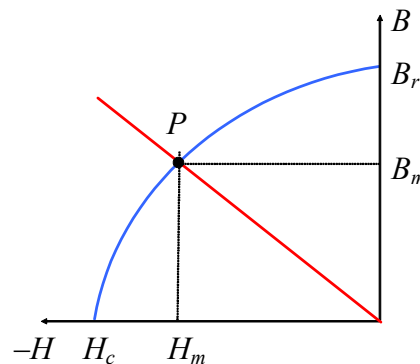


Figure 4-17: Material curve and load line.

5 ELECTROMAGNETIC INDUCTION

5.1 Introduction

Electromagnetic induction is an astonishing effect, which describes the generation of a voltage in a conductor or a circuit. It can happen in two entirely different ways:

- Motional induction, where a conductor moves in a magnetic field.
- Transformer induction, where the circuit is stationary and the field varies with time.

Faraday's law of electromagnetic induction, which relates the generated voltage to the change of flux, can take account of both effects.

Electromagnetic induction is widely misunderstood, and this misunderstanding has resulted in numerous engineering failures. One example is a patent for a whole range of 'improved' DC motors and generators, none of which will work [4].

Motional induction has already been considered in section 2, where it was shown that the voltage induced in a moving conductor is given by:

$$e = Blu \quad [2-1]$$

Although it is the basis of many practical devices, this equation has a deceptive simplicity. Some of the problems that arise from its application are discussed in section 5.3.

5.2 Transformer induction

Faraday's law

Figure 5-1 represents a coil of N turns and cross-sectional area A placed in a magnetic field.

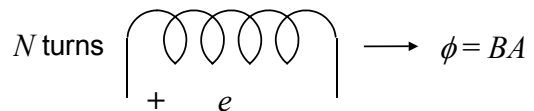


Figure 5-1: Coil in a magnetic field.

Initially the field will be assumed uniform, so that the flux linkage is given by:

$$\lambda = N\phi = NBA \quad (5-1)$$

From equation 4-24, the slope of the line is inversely proportional to the length of the airgap, so the effect of increasing the airgap is to move the operating point P further down the curve.

If the airgap is very small, the load line is nearly vertical. The value of H_m is then very small, and the value of B_m is close to the remanence B_r . This condition is unaffected by the value of B_s in the steel poles, provided that the poles remain unsaturated, so that $H_s l_s$ is still small in comparison with $H_m l_m$ in equation 4-22.

Demagnetisation

Suppose that the airgap in the magnetic circuit is increased, so that the working point moves from P_1 to P_2 in figure 4-18.

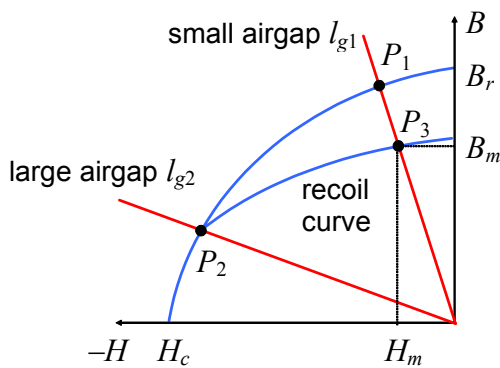


Figure 4-18: Recoil curve.

If the airgap is restored to its original value, the original curve may not be followed. Instead, a new curve known as the *recoil curve* is traversed, as shown in figure 4-18, giving a new working point P_3 with a lower value of B_m . The material has become partly demagnetised. This is a particular problem with the Alnico materials, which are still widely used. With materials such as ceramic ferrite or neodymium iron boron, the recoil curve is indistinguishable from the main curve unless a strong demagnetising field is applied by passing current through a coil.

It is a remarkable fact that a voltage is induced in the coil whenever the flux changes, regardless of what causes it to change. *Faraday's law* of electromagnetic induction states that the voltage e is given by:

$$e = \frac{d\lambda}{dt} = N \frac{d\phi}{dt} \quad [\text{volts, V}] \quad (5-2)$$

Some textbooks put a negative sign in equation 5-4, out of deference to Lenz's law, which states that induced currents act in a direction to oppose the change. However, a positive sign is required for consistency with the normal conventions of circuit theory: see the discussion of self-inductance below.

Worked example 5-1

Find the maximum value of the voltage induced in a coil of 150 turns by a magnetic flux $\phi = \Phi_m \sin \omega t$ if $\Phi_m = 400 \mu\text{Wb}$ and the frequency is 60 Hz.

Solution

The flux linkage is:

$$\lambda = N\phi = N\Phi_m \sin \omega t$$

so the induced voltage is:

$$e = \frac{d\lambda}{dt} = \omega N\Phi_m \cos \omega t = E_m \cos \omega t$$

The maximum value of e is:

$$\begin{aligned} E_m &= \omega N\Phi_m = 2\pi f N\Phi_m \\ &= 2\pi \times 60 \times 150 \times 400 \times 10^{-6} \\ &= 22.6 \text{ V} \end{aligned}$$

Self-inductance

Suppose that the magnetic flux in a coil is the result of a current i in the same coil, as shown in figure 5-2. It will be assumed that the coil does not have a magnetic core, or that the magnetic circuit has a large airgap, so that the flux is proportional to the current.

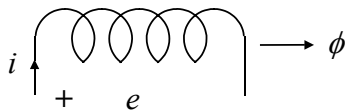


Figure 5-2: Self-inductance.

From equation 3-31 the flux linkage is:

$$\lambda = N\phi = Li \quad [3-31]$$

Faraday's law, equation 5-2, gives the induced voltage as:

$$e = \frac{d\lambda}{dt} = \frac{d}{dt}(Li) = L \frac{di}{dt} + i \frac{dL}{dt} \quad (5-3)$$

If L is constant, this reduces to the defining equation for inductance in circuit theory:

$$e = L \frac{di}{dt} \quad (5-4)$$

If the current i is held constant, but a displacement of part of the system changes the value of L , then equation 5-3 shows that there will be a generated voltage given by:

$$e = i \frac{dL}{dt} = i \frac{dL}{dx} \cdot \frac{dx}{dt} = iu \frac{dL}{dx} \quad (5-5)$$

where u is the velocity corresponding to the rate of change of the displacement x . This is an important property of *variable reluctance* devices, which are considered in section 9.

Mutual inductance

Figure 5-3 shows a pair of adjacent coils, where some flux links the second coil when there is a current i_1 in the first coil. The coils are said to be *coupled* magnetically.

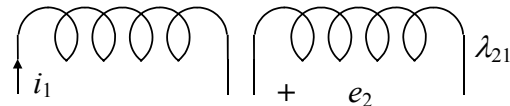


Figure 5-3: Mutual inductance – 1.

Let the flux linkage with the second coil be λ_{21} . As with a single coil, it will be assumed that the coils are wound on non-magnetic cores, or the airgap in the magnetic circuit is large, so that we may put:

$$\lambda_{21} = M_{21}i_1 \quad (5-6)$$

where M_{21} is a constant. In figure 5-4, the roles are reversed: the second coil carries a current i_2 , and we consider the flux linkage with the first coil.

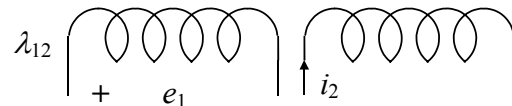


Figure 5-4: Mutual inductance – 2.

Let the flux linkage with the first coil be λ_{12} . With the same assumptions as before, we have:

$$\lambda_{12} = M_{12}i_2 \quad (5-7)$$

where M_{12} is a constant. It may be shown that:

$$M_{21} = M_{12} = M \quad (5-8)$$

where M is termed the *mutual inductance* of the two coils. The reciprocal property of mutual inductance expressed in equation 5-10 does not depend on any similarity of the coils – they can have different shapes, sizes and numbers of turns. Combining equations 5-6, 5-7 and 5-8 gives:

$$M = \frac{\lambda_{12}}{i_2} = \frac{\lambda_{21}}{i_1} \quad (5-9)$$

Equation 5-9 gives two different ways of determining the mutual inductance, and one way is often easier than the other. An application of this principle is given below for coaxial cables.

If the current i_1 in figure 5-3 changes with time, then the voltage e_2 induced in the second coil is given by:

$$e_2 = \frac{d}{dt}(Mi_1) = M \frac{di_1}{dt} \quad (5-10)$$

Similarly, if the current i_2 in figure 5-4 changes with time, then the voltage e_1 induced in the first coil is given by:

$$e_1 = \frac{d}{dt}(Mi_2) = M \frac{di_2}{dt} \quad (5-11)$$

It is assumed that the coils do not move, so M is a constant.

Circuit aspects of coupled coils

Figure 5-5 shows a voltage source connected to each coil, so that currents flow in both coils.

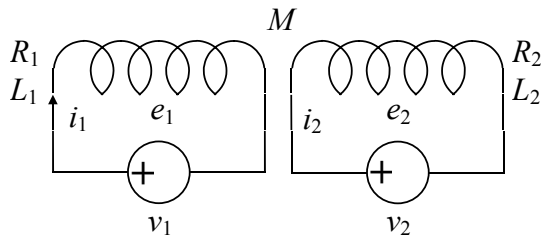


Figure 5-5: Coupled coils.

Kirchhoff's voltage law applied to each coil gives the circuit equations for coupled coils:

$$\begin{aligned} v_1 &= R_1i_1 + e_1 = R_1i_1 + \frac{d}{dt}(L_1i_1 + Mi_2) \\ &= R_1i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \end{aligned} \quad (5-12)$$

$$\begin{aligned} v_2 &= R_2i_2 + e_2 = R_2i_2 + \frac{d}{dt}(L_2i_2 + Mi_1) \\ &= R_2i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \end{aligned} \quad (5-13)$$

It may be shown that the total stored magnetic energy in the system is given by:

$$W_m = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2 \quad (5-14)$$

Coaxial cables

Coaxial cables are widely used for signal connections because they reject interference from external magnetic fields. To show that they have this property, suppose that the source of the magnetic field is a circuit carrying a current i_1 . The voltage induced in the cable is Mdi_1/dt , where M is the mutual inductance between the cable and the circuit. To calculate M , let the cable carry a current i_2 , so that $M = \lambda_{12}/i_2$. Since a perfect coaxial cable produces no external magnetic field (see section 3.5), it follows that $\lambda_{12} = 0$, so $M = 0$. In practice, the conductors may not be perfect coaxial cylinders, so there may be some residual induced voltage.

5.3 Further aspects of induction

Motional induction revisited

Equation 2-1 is sometimes known as the *motional induction formula*.

$$e = Blu \quad [2-1]$$

Its simplicity makes it a trap for the unwary, and it is important to be aware of the restrictions:

- B is the local field at the conductor.
- B is the field of a single source, or the sources of B are not in relative motion.

- u is the velocity relative to the source of B .

The following example shows how the incorrect use of equation 2-1 can give completely wrong answers. Figure 5-6 shows a single-turn coil moving with velocity u in a magnetic field B . One side of the coil is enclosed in a steel tube, which screens it from the magnetic field.

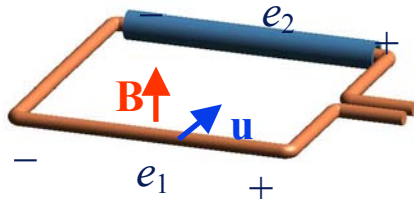


Figure 5-6: Partly screened coil.

If the external field B is uniform in the absence of the steel tube, the effect of the tube is shown by the flux plot in figure 5-7. As an example of the screening effect, if the material of the tube has a relative permeability of 1000, and the inside diameter is 90% of the outside diameter, the flux density inside the tube is about 1% of the value for the uniform field.

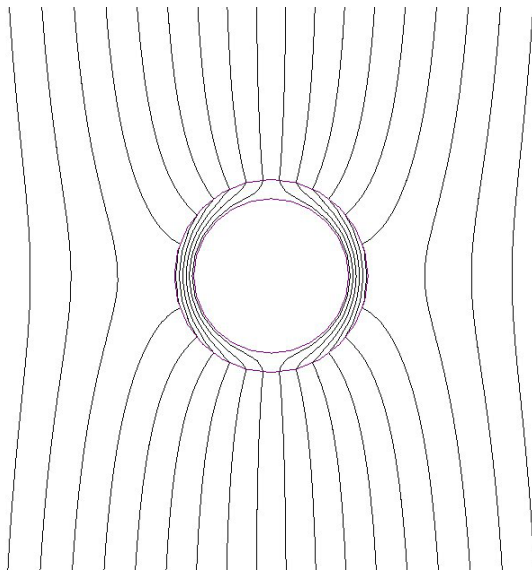


Figure 5-7: Screening effect of a steel tube.

The two coil sides at right angles to the direction of motion are of length l . There is a voltage e_1 induced in unscreened side, given by:

$$e_1 = B_1 l u = B l u \quad (5-15)$$

However, the flux density B_2 in the vicinity of the other coil side is very small, so the direct application of equation 2-1 gives:

$$e_2 = B_2 l u \approx 0 \quad (5-16)$$

The voltage at the terminals of the coil therefore appears to be:

$$e = e_1 - e_2 \approx B l u - 0 = B l u \quad (5-17)$$

If the coil has N turns instead of a single turn, it appears to be possible to generate a voltage given by:

$$e \approx N B l u \quad (5-18)$$

This would provide a neat way of generating electricity by moving a coil of wire through the Earth's magnetic field. However, it will not work for the following reason. The field B_2 is the small difference between two fields: the external field B , and the field B_s created by the steel tube, which is itself magnetised by the external field. Thus:

$$B_2 = B_e - B_s \quad (5-19)$$

The conductor is in motion with respect to the source of B , but it is stationary with respect to the source of B_s , since this is the cylinder that is moving with the conductor. Therefore, it is only the component B that gives rise to a generated voltage, so the voltage e_2 is equal in magnitude to e_1 , and the voltage at the coil terminals is zero.

Similar considerations apply to the non-working inventions mentioned in reference [4].

Transformer induction revisited

In Faraday's law (equation 5-2), the value of the induced voltage depends only on the rate of change of flux linkage.

- The voltage does not depend on the nature of the source of the flux density B .
- The voltage does not depend on the local value of B at the conductor.
- It is possible to have an induced voltage in a circuit even if $B = 0$ in the conductor.

Consider the toroidal coil of section 3.6. Outside the coil, the value of B is very small. If the coil is energised from an AC source, as it is in a toroidal transformer, there will be an alternating magnetic flux in the core. Suppose that a loop of wire encircles the core. The flux linkage with the loop is equal to the flux in the core, so there will be an induced voltage in the loop although the magnetic field is negligible in the wire.

The explanation of this result is that the magnetic field is only half the story. In textbooks on electromagnetic theory [5], it is shown that, whenever a magnetic field is changing with time, there is an associated electric field. Figure 5-8 shows the lines of the electric field \mathbf{E} around a toroidal coil when there is an alternating magnetic flux in the core.

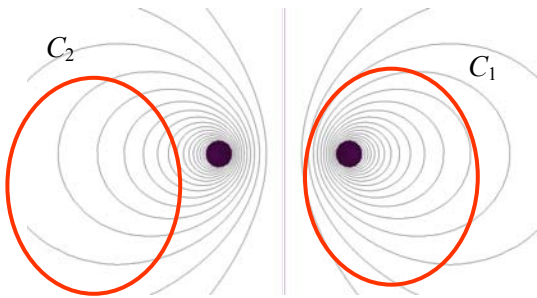


Figure 5-8: Electric field of a toroidal coil.

If a wire completely encircles the core, such as the loop C_1 in figure 5-8, there will be an induced voltage given by:

$$e = \oint_{C_1} \mathbf{E} \cdot d\mathbf{l} = \frac{d\lambda}{dt} \quad (5-20)$$

However, if the wire does not encircle the core, such as the loop C_2 in figure 5-8, then the induced voltage is zero:

$$e = \oint_{C_2} \mathbf{E} \cdot d\mathbf{l} = 0 \quad (5-21)$$

Practical application

The voltage given by equation 5-20 has an important practical consequence. If a toroidal transformer is fastened by a bolt passing through the hole, it is essential that the two ends of the bolt are not connected together by metal parts of the equipment. If they are connected, a conducting loop will be formed around the core, and the induced voltage will circulate a large current around the loop.

General use of Faraday's law

Faraday's law (equation 5-2) holds for any closed circuit, and takes account of both transformer and motional effects.

$$e = \frac{d\lambda}{dt} = N \frac{d\phi}{dt} \quad [5-2]$$

This gives the correct value for the induced voltage, no matter what causes the flux ϕ to change. In a circuit with a fixed boundary, if ϕ does not change, there can be no induced voltage. This is often the quickest way of establishing whether a claimed invention for a new electromechanical device will work. For example, in the coil shown in figure 5-6, there is no change of flux through the coil when it moves in a uniform field, and therefore no voltage will appear at the terminals.

Circuits where the boundary shapes change with time, or those that involve sliding contacts, are notoriously difficult, and require careful analysis to predict the induced voltage correctly [6].

6 TRANSFORMERS

6.1 Introduction

A transformer is a practical application of magnetically coupled coils (see section 5.2). Usually the purpose is to transfer energy from one coil to the other, as in figure 6-1 where energy is transferred from the AC source to the lamp through the space between the coils.

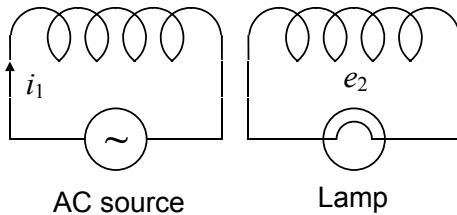


Figure 6-1: Transformer principle.

Transformers are essentially AC devices, because there has to be a change of flux to give an induced voltage in a coil. There are two main reasons for transferring energy in this way:

- To provide electrical isolation between the source and the load.
- To change the voltage and current levels.

The coils are usually placed on a common magnetic core to improve the coupling. Figure 6-2 shows the flux plots for two coupled coils (a) without a core, (b) with an open core, (c) with a closed core.

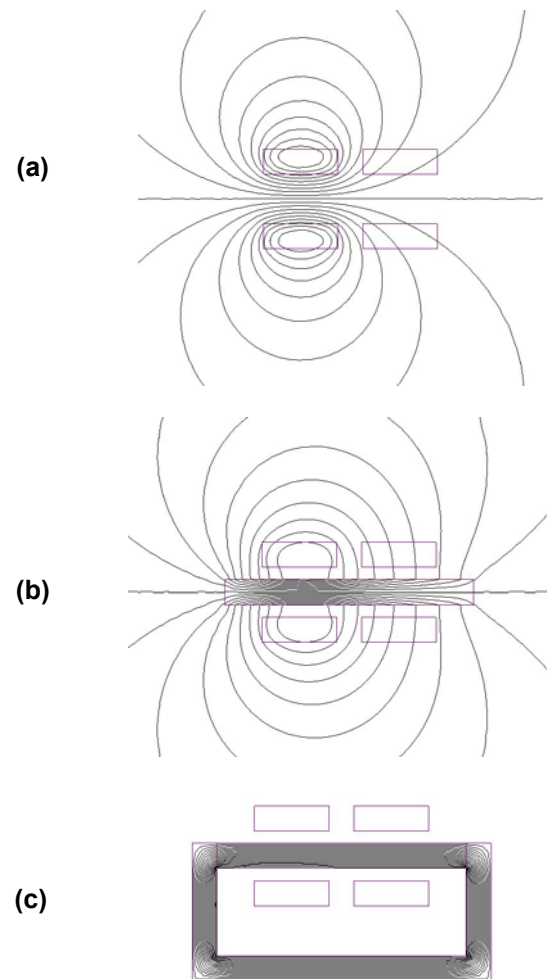


Figure 6-2: Effect of a magnetic core: (a) no core, (b) open core, (c) closed core.

Practical aspects

The cores for high-frequency transformers are often made from magnetically soft ferrites, which are electrical insulators. For power frequencies, the cores are made from an iron alloy such as silicon steel. These materials have better magnetic properties than ferrites, but they are also electrical conductors. If the core were made of solid steel, currents – known as *eddy currents* – would be induced in the core by the alternating magnetic flux. Steel transformer cores are therefore made from thin laminations, insulated from one another, to minimise eddy currents [1, 6].

Small power transformers are commonly made with the shell-type construction shown in figure 6-3, where the coils are wound on the central limb of the core, and the two outer limbs provide symmetrical flux return paths.



Figure 6-3: Shell-type transformers.
(RS Components Ltd)

The centre limb is twice the width of the outer limbs because it carries twice the flux, as shown by the flux plot in figure 6-4.

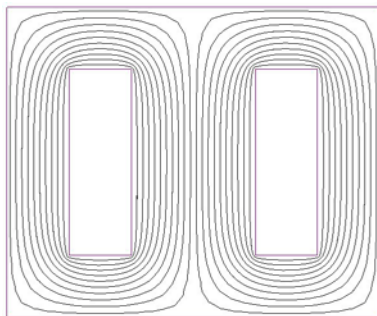


Figure 6-4: Flux plot: shell-type transformer.

Toroidal transformers exploit the remarkable properties of toroidal coils described in section 3.6. Although they are more expensive than shell-type transformers, the performance is better. They are used in high-quality electronic equipment and for instrument transformers (see section 6.3) where measurement accuracy is important. Typical toroidal transformers are shown in figure 6-5.



Figure 6-5: Toroidal transformers.
(RS Components Ltd)

6.2 Transformer principles

The action of a transformer is most easily understood if the two coils are wound on opposite sides of a magnetic core, as shown in the model of figure 6-6. This form is used for some low-cost transformers, but the magnetic coupling is not as good as with the shell-type construction.

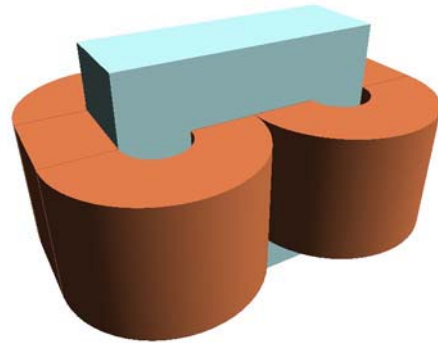


Figure 6-6: Core-type transformer.

Figure 6-7 is a schematic representation of the transformer. It will be assumed that the coupling is perfect: the same magnetic flux ϕ passes through each turn of each coil.

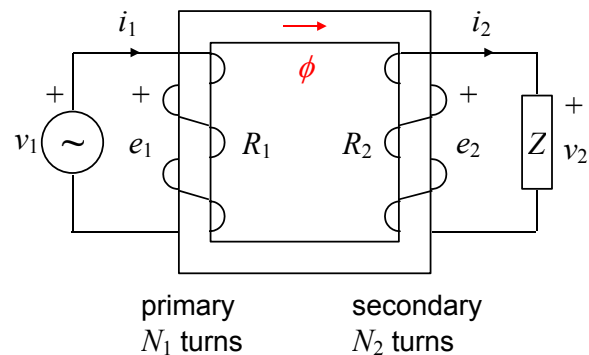


Figure 6-7: Transformer with source and load

The coil connected to the source is termed the *primary*, and the coil connected to the load is termed the *secondary*. It is usual to refer to the coils as *windings*.

Voltage relationships

Faraday's law (equation 5-2) applied to the two windings gives the following expressions for the voltages induced in the windings:

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt} \quad (6-1)$$

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi}{dt} \quad (6-2)$$

Kirchhoff's voltage law applied to the two windings gives:

$$v_1 = e_1 + R_1 i_1 = N_1 \frac{d\phi}{dt} + R_1 i_1 \quad (6-3)$$

$$v_2 = e_2 - R_2 i_2 = N_2 \frac{d\phi}{dt} - R_2 i_2 \quad (6-4)$$

The sign difference arises from the reference directions for current in the two windings.

If the resistances R_1 and R_2 are negligible, then equations 6-3 and 6-4 become:

$$v_1 \approx N_1 \frac{d\phi}{dt} \quad (6-5)$$

$$v_2 \approx N_2 \frac{d\phi}{dt} \quad (6-6)$$

Dividing these equations gives the important result:

$$\frac{v_1}{v_2} \approx \frac{N_1}{N_2} \quad (6-7)$$

Thus, the secondary voltage can be made larger or smaller than the primary voltage by changing the ratio of the numbers of turns on the two windings.

Voltage transformation is one of the most common uses of transformers, on a large scale in electrical power transmission and distribution, and on a small scale in the power supplies for electronic equipment.

Sinusoidal operation

If the voltage source is sinusoidal, then the core flux will also be sinusoidal, so we may put:

$$\phi = \Phi_m \sin \omega t \quad (6-8)$$

Substituting this expression in equation 6-5 gives:

$$v_1 = N_1 \frac{d\phi}{dt} = N_1 \omega \Phi_m \cos \omega t = V_{1m} \cos \omega t \quad (6-9)$$

Thus the maximum primary voltage is:

$$V_{1m} = N_1 \omega \Phi_m = 2\pi f N_1 \Phi_m = 2\pi f N_1 A B_m \quad (6-10)$$

where A is the cross-sectional area of the core and B_m is the maximum flux density in the core.

We can deduce some of the limits of transformer operation from equation 6-10. It is normal practice to design a transformer with B_m close to the saturation value for the core material when V_1 and f have their normal rated values. A typical value for B_m is 1.4 T for the silicon steel characteristic in figure 6-8.

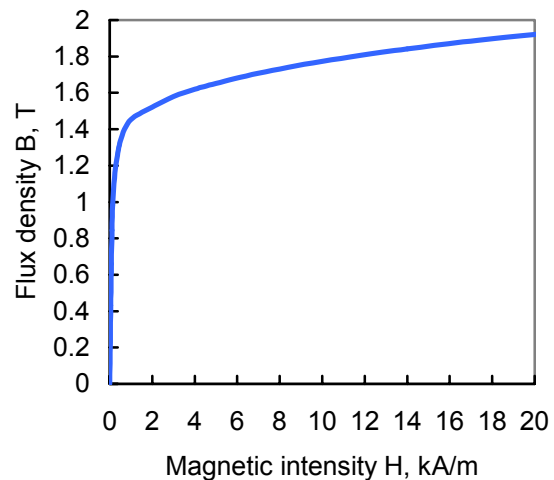


Figure 6-8: Silicon transformer steel.

From equation 6-10, if f is increased or V_1 is reduced, then the value of B_m will fall. The transformer will be under-run, but this will not cause any problems.

However, if f is reduced below the design value, or V_1 is increased above the design value, then B_m will be required to increase. This will take the core into saturation. The magnetisation curve in figure 6-8 shows that there will be a very large increase in the value of H , which in turn will require a large current in the primary. The resulting $I^2 R$ power loss in the primary winding will cause the temperature to rise, which may damage or destroy the transformer.

Current relationships

The relationship between the primary and secondary currents can be found by considering the magnetic circuit of the transformer. From equation 4-8 we have:

$$F = N_1 i_1 - N_2 i_2 = R\phi \quad (6-11)$$

The negative sign arises from the reversed reference direction of the secondary current. R is the reluctance of the core, given by:

$$R = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_0 A} \quad (6-12)$$

If the relative permeability μ_r is large, $R\phi$ will be small in comparison with $N_1 i_1$ or $N_2 i_2$, so equation 6-11 becomes:

$$N_1 i_1 - N_2 i_2 \approx 0 \quad (6-13)$$

Thus:

$$N_1 i_1 \approx N_2 i_2 \quad (6-14)$$

Equation 6-14 may be expressed in a similar form to the voltage relationship in equation 6-7:

$$\frac{i_1}{i_2} \approx \frac{N_2}{N_1} \quad (6-15)$$

Observe that the current is related to the turns ratio in the opposite way to the voltage. If the secondary voltage is smaller than the primary voltage, then the current is larger, and *vice versa*. If losses are ignored, then the power input to the primary will be equal to the power output from the secondary.

Ideal transformer element

If the primary and secondary windings have zero resistance, and the magnetic core has zero reluctance, then the approximate equalities in equations 6-7 and 6-15 become exact equalities. This leads to the concept of an *ideal transformer element*, to accompany the other ideal elements of circuit theory. Figure 6-9 shows a circuit symbol for the ideal transformer element.

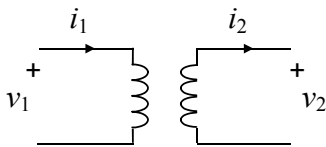


Figure 6-9: Ideal transformer element

The voltage and current relationships are as follows, in the time and frequency domains:

Time domain	Frequency domain
$\frac{v_2}{v_1} = \frac{N_2}{N_1}$	$\frac{V_2}{V_1} = \frac{N_2}{N_1} \quad (6-16)$

$\frac{i_1}{i_2} = \frac{N_2}{N_1}$	$\frac{I_1}{I_2} = \frac{N_2}{N_1} \quad (6-17)$
-------------------------------------	--

The ideal transformer is a useful guide to the behaviour of real transformers, and forms the basis of a circuit model described below. The following properties of the ideal transformer may be deduced from equations 6-16 and 6-17:

- The voltage transformation is independent of the current, and *vice versa*.
- If the secondary is short-circuited, so that $v_2 = 0$, the primary terminals appear to be short-circuited since $v_1 = 0$.
- If the secondary is open-circuited, so that $i_2 = 0$, the primary terminals appear to be open-circuited since $i_1 = 0$.
- The output power is equal to the input power, so there is no power loss in the element.

Circuit model of real transformers

In a real transformer, the winding resistances and the core reluctance are not zero, and the magnetic coupling is not perfect. In addition, power will be lost in the core because of eddy currents and hysteresis in the magnetic material [1, 4]. All of these effects can be represented by the equivalent circuit [4] shown in figure 6-10.

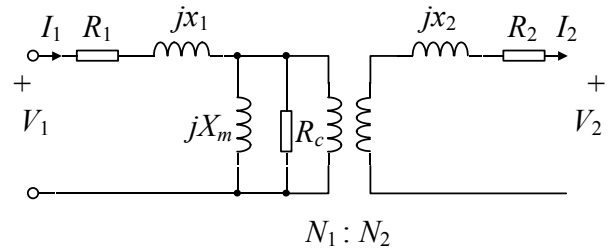


Figure 6-10: Transformer equivalent circuit.

This circuit is based on the ideal transformer element, with additional circuit elements to represent the imperfections. The resistances R_1 and R_2 represent the physical resistances of the windings, and R_c represents the power lost in the core. The reactance X_m , known as the *magnetising reactance*, allows for the current required to magnetise the core when the reluctance is not zero.

Reactances x_1 and x_2 , known as *leakage reactances*, account for the imperfect magnetic coupling between the primary and the secondary.

6.3 Current transformers

Instrument transformers are special transformers for extending the range of measuring instruments. There are two basic types: voltage transformers for measuring high voltages, and current transformers for measuring high currents. Using transformers for voltage measurement is similar in principle to the ordinary use of transformers to change voltage levels, so it will not be considered further. Current transformers, on the other hand, need special consideration. These are usually toroidal transformers with high-quality core material.

Figure 6-11 shows a load connected to a source. The primary of a current transformer is in series with the load, and the secondary is connected to a meter.

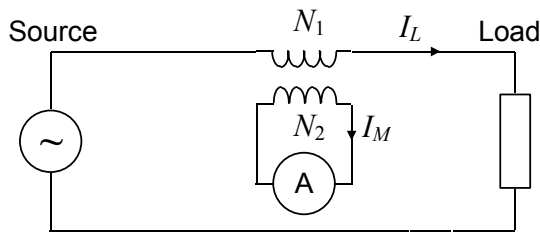


Figure 6-11: Use of a current transformer.

Equation 6-15 gives:

$$I_M \approx \frac{N_1}{N_2} I_L \text{ or } I_L \approx \frac{N_2}{N_1} I_M \quad (6-18)$$

Thus, the unknown current I_L in the load can be determined from the measured current I_M . With a well-designed current transformer, the error in the current measurement from equation 6-18 is typically less than 1%.

In many applications, the current-transformer primary is permanently connected in the main circuit, where the current may be very large. The secondary current is usually low – typically, no more than 5 A – and the secondary voltage is also low because the meter resistance is small. It is normal practice to connect one terminal of the meter to earth, so that it is safe to handle the terminals.

Worked example 6-1

A current transformer has 10 turns on the primary and 200 turns on the secondary. The secondary is

connected to a meter with a resistance of 0.5Ω , and the secondary current is 5 A. Find the current in the primary and the voltage across the primary terminals. Assume that the transformer is ideal.

If the transformer has a toroidal core with a cross-section of $20 \text{ mm} \times 20 \text{ mm}$, and the frequency is 50 Hz, determine the maximum value of the flux density in the core.

Solution

The secondary terminal voltage is:

$$V_2 = RI_2 = 0.5 \times 5.0 = 2.5 \text{ V}$$

The primary current is:

$$I_1 = \frac{N_2}{N_1} I_2 = \frac{200}{10} \times 5.0 = 100 \text{ A}$$

The primary terminal voltage is:

$$V_1 = \frac{N_1}{N_2} V_2 = \frac{10}{200} \times 2.5 \text{ V} = 125 \text{ mV}$$

From equation 6-10, the maximum flux density in the core is:

$$B_m = \frac{V_{1m}}{2\pi f N_1 A} = \frac{\sqrt{2} \times 125 \times 10^{-3}}{2\pi \times 50 \times 10 \times (20 \times 10^{-3})^2} \text{ T} \\ = 0.141 \text{ T}$$

Open-circuit secondary

Problems arise if it is required to disconnect the meter while current continues to flow in the main circuit. Consider the circuit of figure 6-12, where the load is purely resistive and a switch has been substituted for the meter.

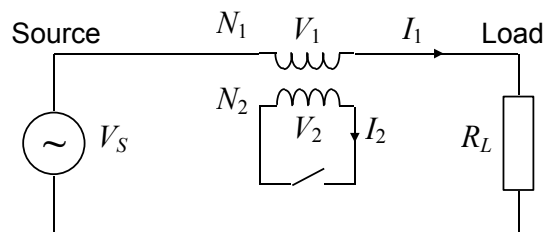


Figure 6-12: Open-circuit secondary.

With the switch closed, $V_2 = 0$, so $V_1 \approx 0$, and the primary current is determined by the source voltage and the load resistance:

$$I_1 = \frac{V_s}{R_L} \quad (6-19)$$

The secondary current is thus:

$$I_2 \approx \frac{N_1}{N_2} I_1 \quad (6-20)$$

With the switch open, the situation is complex. Suppose initially that the transformer can be regarded as ideal. With the switch open, $I_2 = 0$, and therefore $I_1 = 0$. With no current in the load, the transformer primary voltage V_1 must be equal to the source voltage V_s .

In practice, this cannot happen. In worked example 6-1, the primary voltage V_1 is only 125 mV in normal operation, and the core flux density is then 0.141 T. If the source voltage V_s is 400 V, it would require a core flux density of 451 T to make V_1 equal to V_s . Since the core material saturates at about 1.6 T, this is impossible.

Opening the secondary switch will therefore not make any significant difference to the primary current, which is still determined by the load resistance R_L . However, the conditions in the transformer are very far from normal. Consider the magnetic circuit equations. With the switch closed, we have:

$$N_1 i_1 - N_2 i_2 \approx 0 \quad [6-13]$$

With the switch open, there is no secondary MMF to balance the primary MMF $N_1 I_1$, so the core material is subjected to a large alternating H field given by equation 4-2:

$$H = \frac{N_1 i_1}{l} = \frac{N_1 I_{1m} \sin \omega t}{l} \quad (6-21)$$

where l is the mean length of the magnetic core. The core material is taken well into saturation, so the flux waveform is almost a square wave. This can result in dangerously high voltage pulses being induced in the secondary, which can damage the transformer insulation and present a hazard to the operator. See the numerical example on page 32.

Safe operation

If it is necessary to disconnect the secondary load without interrupting the primary current, this can be done safely by including a switch in parallel with the load as shown in figure 6-13. The switch must be closed before disconnecting the load, so that the secondary current I_2 continues to flow through the switch.

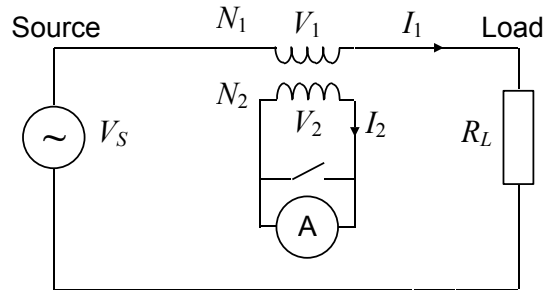


Figure 6-13: Safe operation.

Numerical example

Suppose that the current transformer has a toroidal core with a cross-section of 20 mm × 20 mm and a mean diameter of 100 mm. There are 10 turns on the primary, 200 turns on the secondary, the RMS current in the primary is 89 A, and the frequency is 50 Hz. The magnetic material is silicon steel, with the magnetisation data given in section 12-1. From equation 6-21, the maximum value of H is:

$$H_m = \frac{N_1 I_{1m}}{\pi D} = \frac{10 \times \sqrt{2} \times 89}{\pi \times 100 \times 10^{-3}} = 4000 \text{ A/m} \quad (6-22)$$

The period of the waveform is:

$$T = \frac{1}{f} = \frac{1}{50} \text{ s} = 20 \text{ ms} \quad (6-23)$$

Figure 6-14(a) shows the waveform of H , which is proportional to the sinusoidal primary current. This is applied to the magnetisation curve in figure 6-14(b). It takes the magnetic material well into saturation, so the resulting waveform of B , shown in figure 6-14(c), is almost a square wave. The red dots on these graphs represent successive instants of time during the first quarter cycle of the current waveform.

The voltage induced in the secondary is:

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi}{dt} = N_2 A \frac{dB}{dt} \quad (6-24)$$

where A is the cross-sectional area of the core.

For most of the time, B is almost constant, so dB/dt is small. However, when the sinusoidal current changes from negative to positive, B suddenly changes from negative saturation to positive saturation, and the resulting dB/dt is very large. There is a similar effect at the negative-going transition. Figure 6-14(d) shows a graph of dB/dt , obtained by numerical differentiation of the graph

of B . From this graph, the maximum value of dB/dt is 14.4 T/ms. The maximum induced voltage in the secondary is therefore:

$$\begin{aligned}
 e_{2\max} &= N_2 A \frac{dB}{dt} \\
 &= 200 \times (20 \times 10^{-3})^2 \times 14.4 \times 10^3 \quad (6-25) \\
 &= 1150 \text{ V}
 \end{aligned}$$

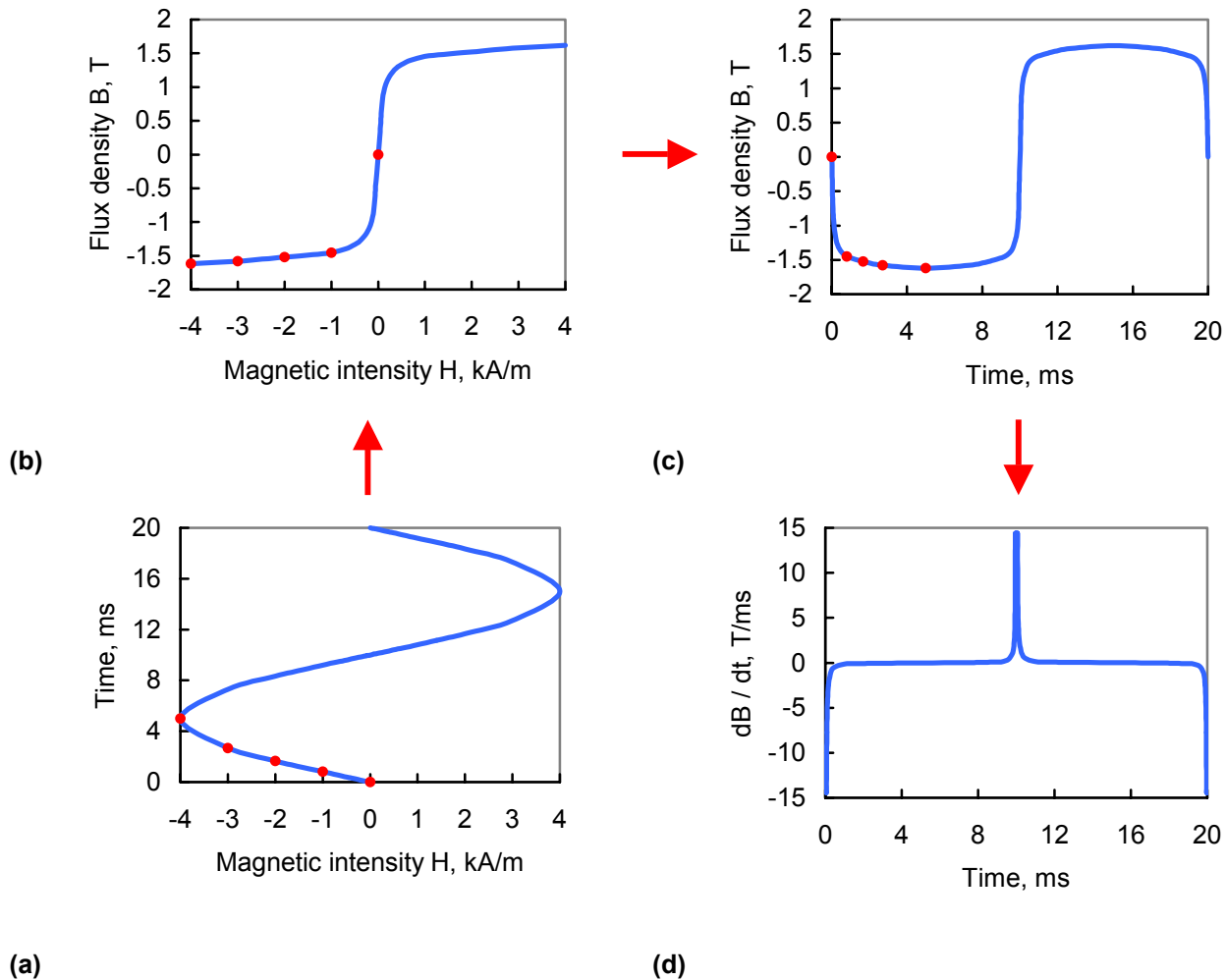


Figure 6-14: Current transformer with open-circuit secondary.
(a) H waveform, (b) magnetisation curve, (c) B waveform, (d) dB/dt waveform.

6.4 Induction heating

Transformer cores are laminated to minimise eddy currents. If this were not done, the core would get very hot. Eddy-current heating is an unwanted effect in transformers, but it is a useful industrial process for localised heating of steel components. This is termed *induction heating*. Figure 6-15 shows a simple example, where a coil of N turns surrounds a steel bar.



Figure 6-15: Eddy-current heating.

If the coil is supplied with alternating current at the normal mains frequency, an alternating magnetic flux will be set up in the bar, and induced currents will flow in circular paths. These currents are not uniformly distributed, but are concentrated in a layer near the surface. This is known as *skin effect* [4], and the effective depth of the layer is the skin depth. For ordinary mild steel, the skin depth at 50 Hz is about 1 mm.

The immediate heating effect of eddy currents is confined to the surface layer, which can be heated very rapidly if the coil current is high. This can be useful for heat treatment of mechanical components.

Skin effect has a further consequence: the magnetic flux density is not uniform, but is also confined to the region near the surface. It follows that there is very little field in the interior of the bar if the radius is several times greater than the skin depth. Consequently, the induction heater operates as a current transformer, where the current in the surface layer is N times the current in the coil. This result may be proved by applying Ampère's circuital law to the contour $abcd$ in figure 6-16.

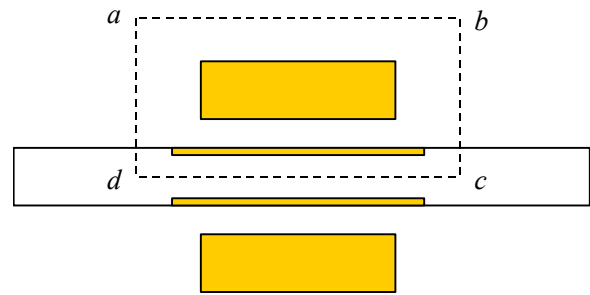


Figure 6-16: Contour for Ampère's circuital law.

If i_1 is the current in the coil and i_2 is the current in the bar, then:

$$\sum i = Ni_1 - i_2 = \oint \mathbf{H} \cdot d\mathbf{l} \approx 0 \quad (6-26)$$

In equation 6-24, the integral is approximately zero for the following reasons. For the portions ab , bc and da in air, H is small because of the flux-guiding effect of the bar. For the portion cd in the bar, H is also small because of skin effect. Thus:

$$i_2 \approx Ni_1 \quad (6-27)$$

6.5 Induction levitation

Figure 6-17 shows the jumping ring device. It comprises a vertical laminated steel bar with a coil at the bottom, and an aluminium ring encircling the bar.



Figure 6-17: Jumping ring device.

When the coil is energised from an AC source, the ring jumps in the air. To explain this, figure 6-18 shows the magnetic flux plot.

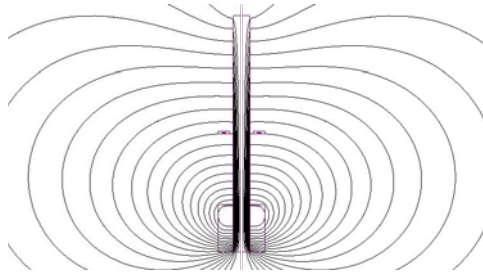


Figure 6-18: 2D flux plot for the jumping ring.

The alternating magnetic flux in the bar induces a current in the ring, and this current interacts with the magnetic field outside the bar to give a vertical force on the ring. As the ring moves up the bar, the force diminishes because there is less flux in the bar, until the electromagnetic force just balances the weight and the ring appears to float. This is a simple demonstration of induction levitation.

Practical levitator

Figure 6-19 shows a practical arrangement for generating large levitation forces by induction.

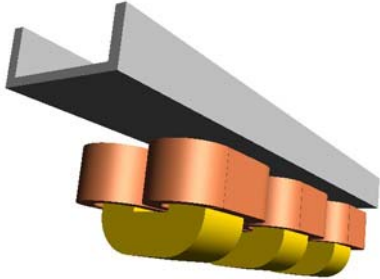


Figure 6-19: Practical induction levitator.

An aluminium channel is placed over an array of U-core electromagnets energised from an AC source. As with the jumping ring, the alternating magnetic field induces currents in the channel, which in turn interact with the field to give a force. Figure 6-20 shows the magnetic field of one electromagnet. Figure 6-21 shows a colour map of the induced current in the channel, where the colour range from blue to red represents the range of minimum to maximum current.

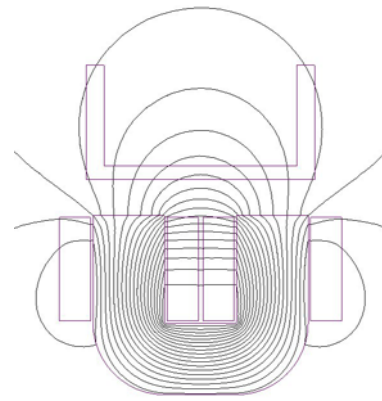


Figure 6-20: Flux plot for one electromagnet.

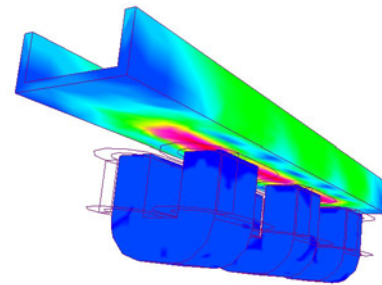


Figure 6-21: Induced current in the channel.

The current is most intense in the middle of the channel, between the poles of the electromagnets, and this current interacts with the horizontal component of the magnet field to give a vertical force.

Figure 6-22 shows an application of this induction levitator in a demonstration track for electromagnetic acceleration of spacecraft. It uses a linear induction motor for propulsion (see section 10.3), so there is no mechanical contact between the moving carrier and the track.



Figure 6-22: Electromagnetic accelerator track. (NASA Marshall Space Flight Centre)

7 MOVING-COIL DEVICES

7.1 Introduction

Moving-coil devices make use of two effects: the force on a current in a magnetic field, and the voltage induced when a conductor moves in a magnetic field. A common example is the loudspeaker shown in figure 7-1, where the coil moves a cone-shaped diaphragm to generate sound waves.



Figure 7-1: Moving-coil loudspeaker.

Figure 7-2 shows a computer model of the magnet and coil structure for a loudspeaker. A ring of permanent-magnet material is the source of the magnetic field, with steel plates and a steel centre pole to guide the flux into the airgap where the coil moves. The coil carries current in a circumferential direction, at right angles to the radial magnetic field in the airgap (figure 7-3), giving a force directed along the axis.

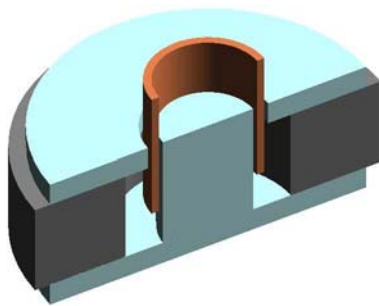


Figure 7-2: Model of magnet and coil.

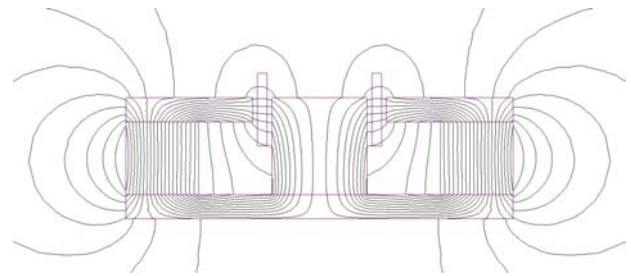


Figure 7-3: Flux plot of a loudspeaker magnet.

Other moving-coil applications include:

- Dynamic microphones
- Vibration generators and sensors
- Precision balances
- Analogue meters
- Head positioning in computer disk drives

7.2 Moving-coil transducers

Loudspeakers and vibration generators are examples of *transducers*, which convert physical quantities of one kind into another. The basic equations are those already derived for a conductor in a magnetic field:

$$e = Blu \quad [2-1]$$

$$f = Bli \quad [2-2]$$

The quantity Bl is known as the *transducer constant*. From equation 2-2, the units are usually given as newtons/ampere (N/A). These equations describe the conversion of electrical quantities into mechanical, and *vice versa*:

Electrical		Mechanical
Current	→	Force
Voltage	←	Velocity

Worked example 7-1

A moving-coil loudspeaker has a transducer constant $Bl = 10 \text{ N/A}$. If the coil carries a sinusoidal current with an RMS value of 2.0 A , find the maximum value of the force on the coil. If the coil motion is sinusoidal and the maximum velocity is 3.0 m/s , find the RMS value of the induced voltage.

Solution

The maximum force is:

$$F_m = BlI_m = Bl\sqrt{2}I = 10.0 \times \sqrt{2} \times 2.0 = 28.3 \text{ N}$$

The RMS voltage is:

$$V = BlU = Bl \frac{U_m}{\sqrt{2}} = 10.0 \times \frac{3.0}{\sqrt{2}} = 21.2 \text{ V}$$

Ideal transducer

It is useful to introduce the concept of an ideal moving-coil transducer, shown in figure 7-4, which has some similarities with the ideal transformer element (section 6.2).

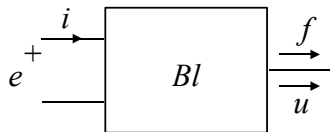


Figure 7-4: Ideal moving-coil transducer.

Multiplying equation 2-1 by the current i , and equation 2-2 by the velocity u , gives:

$$ei = Blui \quad (7-1)$$

$$fu = Bliu \quad (7-2)$$

Since the right-hand sides are identical, the mechanical power output fu is equal to the electrical power input ei . If the sign of i is reversed, then the sign of f also reverses, and the power flow is reversed. Similar considerations apply to u and e . Thus, the moving-coil transducer is a reversible converter of electrical energy to mechanical: a process known as *electromechanical energy conversion*. The following properties should be noted:

- There is no power loss in an ideal transducer.
- If the coil is clamped, so that $u = 0$, the input terminals appear to be short-circuited because $e = 0$.

Model of a real transducer

Practical transducers can be modelled by a combination of an ideal transducer and other elements representing the electrical and mechanical properties of the system [7]. Figure 7-5 shows a simple system model, which includes the resistance R_c and inductance L_c of the coil. The mechanical part is represented by a mass M , a spring with compliance K , and viscous damping with a coefficient D .

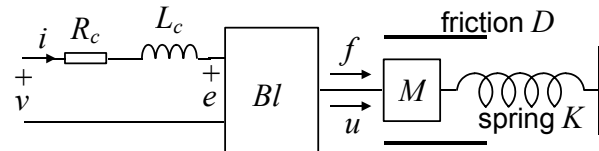


Figure 7-5: Simple system model.

If x is the displacement of the moving part from its rest position, the velocity is given by:

$$u = \frac{dx}{dt} \quad (7-3)$$

The spring restoring force f_s and the damping force f_d are given by:

$$f_s = \frac{x}{K} \quad (7-4)$$

$$f_d = Du \quad (7-5)$$

Applying Newton's second law to the mechanical system gives:

$$f - f_s - f_d = M \frac{du}{dt} \quad (7-6)$$

Substituting for f_s and f_d gives:

$$\begin{aligned} f &= \frac{x}{K} + Du + M \frac{du}{dt} \\ &= \frac{1}{K} \int u dt + Du + M \frac{du}{dt} \end{aligned} \quad (7-7)$$

Equation 7-7 can be expressed in terms of the electrical variables e and i by making the following substitution from equation 2-1:

$$u = \frac{e}{Bl} \quad (7-8)$$

This gives:

$$Bli = f = \frac{1}{Bl} \left(\frac{1}{K} \int e dt + De + M \frac{de}{dt} \right) \quad (7-9)$$

Thus:

$$i = \frac{1}{(Bl)^2 K} \int e dt + \frac{D}{(Bl)^2} e + \frac{M}{(Bl)^2} \frac{de}{dt} \quad (7-10)$$

Consider the parallel combination of inductance, capacitance and resistance shown in figure 7-6.

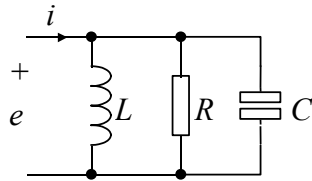


Figure 7-6: Parallel LCR circuit.

The voltage equation for this circuit is:

$$i = \frac{1}{L} \int e dt + \frac{e}{R} + C \frac{de}{dt} \quad (7-11)$$

This is identical to equation 7-10 if we make the following substitutions:

$$L = (Bl)^2 K \quad (7-12)$$

$$R = \frac{(Bl)^2}{D} \quad (7-13)$$

$$C = \frac{M}{(Bl)^2} \quad (7-14)$$

This establishes the important result that, from the electrical terminals, the mechanical system behaves as a parallel combination of inductance, capacitance and resistance. From equation 7-14, this is termed the *mass-capacitance analogy* [7].

The complete system model can therefore be represented in purely electrical terms, as shown in figure 7-7.

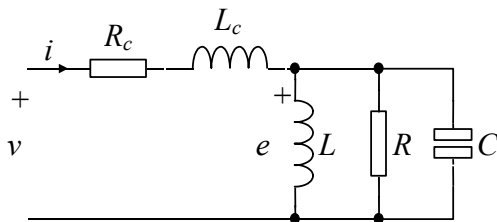


Figure 7-7: Electrical system model.

The parallel *LC* combination will exhibit *resonance* at a frequency ω_0 such that the reactances are equal:

$$\omega_0 L = \frac{1}{\omega_0 C} \quad (7-15)$$

The resonant frequency is thus:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{MK}} \quad (7-16)$$

The second expression in equation 7-16 is obtained by substituting for *L* and *C* from equations 7-12 and 7-14.

With large moving-coil transducers, the resonant frequency is low and the coil inductance *L_c* can be neglected for operation in the region of resonance. The frequency-domain equivalent circuit then takes the form shown in figure 7-8.

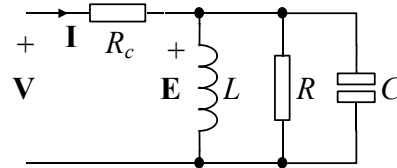


Figure 7-8: Low-frequency equivalent circuit.

The input impedance is given by:

$$\begin{aligned} \mathbf{Z}_{in} &= \frac{\mathbf{V}}{\mathbf{I}} = R_{in} + jX_{in} \\ &= R_c + \frac{1}{\frac{-j}{\omega L} + \frac{1}{R} + j\omega C} \end{aligned} \quad (7-17)$$

Below resonance, when $\omega < \omega_0$, the impedance is inductive since we have:

$$\frac{1}{\omega L} > \omega C \quad (7-18)$$

Above resonance, when $\omega > \omega_0$, the impedance is capacitive since we have:

$$\frac{1}{\omega L} < \omega C \quad (7-19)$$

At resonance, the impedance is purely resistive:

$$\mathbf{Z}_{in} = R_c + R \quad (7-20)$$

Figures 7-9 and 7-10 show the real and imaginary parts of the input impedance (R_{in} and X_{in}), plotted against the frequency ratio ω / ω_0 , for a typical transducer. This has important consequences for the power amplifier driving the transducer, which must be able to cope with this impedance variation without instability or distortion.

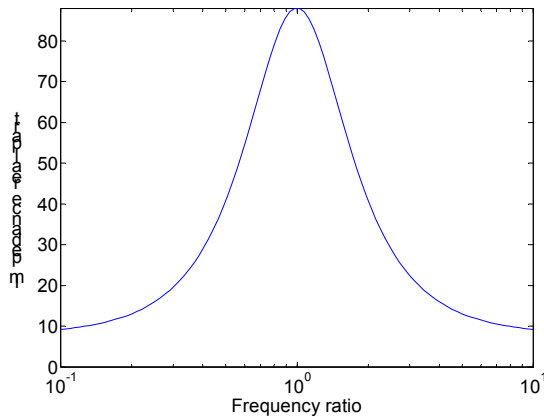


Figure 7-9: Real part of the input impedance.

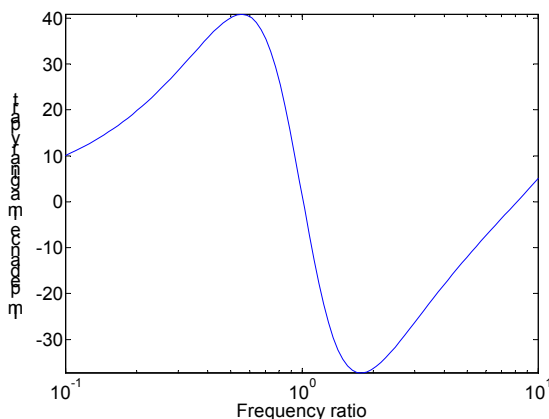


Figure 7-10: Imaginary part the input impedance.

Loudspeakers

A microphone responds to sound waves reaching the diaphragm by converting the coil velocity into a voltage. This voltage is amplified and the resulting signal applied to the loudspeaker coil. The loudspeaker should do the opposite of the microphone: convert the voltage into a corresponding velocity of the cone.

Consider the behaviour of a loudspeaker represented by the equivalent circuit of figure 7-8. The voltage E is proportional to the coil velocity (from equation 2-1). If this voltage is proportional to the applied voltage V , the loudspeaker will

perform as intended. We need to determine the conditions that will give this result.

The parallel combination of L , R and C represents the moving system, so the corresponding impedance is termed the *motional impedance* Z_m . The voltage magnitude E is given by:

$$E = Z_m I \quad (7-21)$$

where I is the magnitude of the coil current. Near the resonant frequency, Z_m will be large in comparison with the coil resistance R_c , so the voltage drop $R_c I$ will be small in comparison with the E , and $E \approx V$. However, at frequencies well above resonance, the impedance Z_m falls. The current I increases, and the voltage drop $R_c I$ becomes significant. Consequently, E becomes much smaller than V , and the loudspeaker sound output diminishes. If the frequency is high enough, the reactance $1 / \omega C$ will be much lower than the resistance R or the reactance ωL , so the R and L elements can be removed, giving the approximate equivalent circuit shown in figure 7-11.

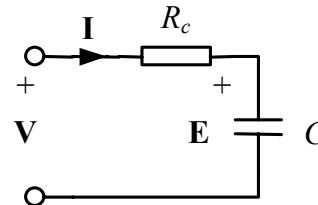


Figure 7-11: Approximate equivalent circuit.

The cut-off frequency is the value that makes the reactance of C equal to the resistance R_c :

$$R_c = \frac{1}{\omega_c C}, \text{ giving } f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi R_c C} \quad (7-22)$$

Above the cut-off frequency, the reactance of C is smaller than the value of R_c , so the voltage E falls as the frequency increases. The ratio E / V is:

$$\frac{E}{V} = \left| \frac{\frac{1}{j\omega C}}{R_c + \frac{1}{j\omega C}} \right| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \quad (7-23)$$

A graph of E / V against the frequency ratio f / f_c is shown in the blue curve of figure 7-12.

If a loudspeaker is designed for the required power output at low frequencies, the resulting mass of the moving system will give a large value for C , and a correspondingly low cut-off frequency – typically below 1 kHz.

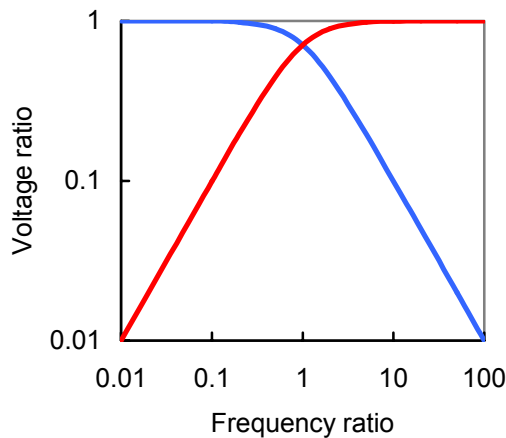


Figure 7-12: Loudspeaker frequency response.
 — LF driver — HF driver

To compensate for this, most hi-fi loudspeakers incorporate two drivers: one for low frequencies and another for high frequencies. A *crossover network* is used to channel low frequencies to one driver and high frequencies to the other [8]. The simplest arrangement is to connect the low-frequency driver directly to the amplifier output, and to connect the high-frequency driver through a capacitor, as shown in figure 7-13.

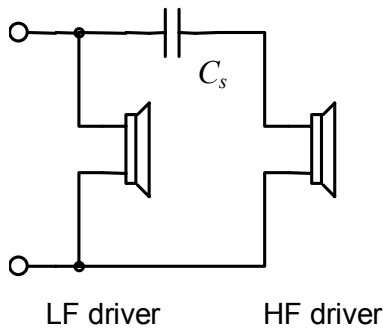


Figure 7-13: Simple crossover circuit.

At low frequencies, the high-frequency driver can be represented by a resistance R_h . If E_h is the voltage across this driver, the ratio E_h / V is:

$$\frac{E_h}{V} = \left| \frac{R_h}{R_h + \frac{1}{j\omega C_s}} \right| = \frac{1}{\sqrt{1 + \left(\frac{f_c'}{f}\right)^2}} \quad (7-24)$$

The cutoff frequency is given by:

$$f_c' = \frac{\omega_c}{2\pi} = \frac{1}{2\pi R_h C_s} \quad (7-25)$$

If C_s is chosen so that $f_c' = f_c$, the graph of E_h / V against the frequency ratio is shown in the red curve of figure 7-12. The graph shows how one driver takes over from the other as the frequency changes. This technique is employed in successful commercial designs such as the Epos ES-11 loudspeaker.

8 DC MACHINES

8.1 Introduction

DC motors convert electrical energy into mechanical energy; DC generators do the opposite. Both types of DC machine have the same basic structure, and they use a similar principle to moving-coil devices.

Figure 8-1 shows a simplified model of a DC machine. The active conductors are placed on the surface of a steel cylinder, and an electromagnet creates a radial magnetic field in the airgap as shown in the flux plot of figure 8-2.

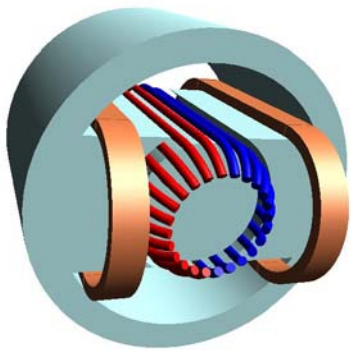


Figure 8-1: DC machine model.

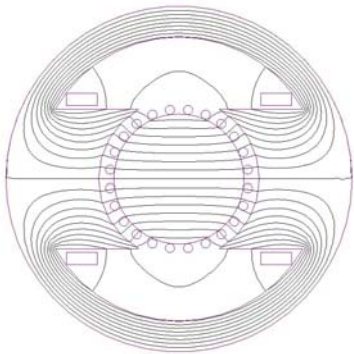


Figure 8-2: DC machine flux plot.

The rotating part with the active conductors is termed the *armature*, and the electromagnet is the *field system*. As the armature rotates, connections are made to armature conductors through sliding contacts on a *commutator*, which is not shown in figure 8-1. The function of the commutator is to route current through the armature conductors in a particular way. All the conductors under one field pole carry current in one direction, and all the conductors under the other pole carry current in the

opposite direction. This is shown by the red and blue colours in figure 8-1.

Figure 8-3 shows the armature and commutator of a typical small DC motor. The armature conductors take the form of coils, which are placed in semi-closed slots in the steel core to reduce the size of the airgap. As in transformers, the core is laminated to minimise eddy currents. The ends of the coils are taken to segments of the copper commutator, which is visible at the right-hand side of the picture. Sliding contacts, known as *brushes*, supply current to the commutator. For a description of a method of interconnecting the coils and commutator segments, see reference [1].



Figure 8-3: DC motor armature.

Putting the coils in slots has an important practical consequence: most of the electromagnetic force acts on the steel core of the armature instead of the conductors themselves. Nevertheless, the force is the same as would have been exerted on a conductor in the airgap [4].

The field system of a DC machine can be an electromagnet, as in figure 8-1. This is known as a *wound field*, and it is always used in large machines. Small machines often use permanent magnets instead of a wound field.

DC generators

DC generators are virtually obsolete as power sources – they have been superseded by AC generators and rectifiers. Nowadays the only significant application for DC generators is in speed sensing. DC *tachogenerators* are precision DC generators designed for use as speed sensors in control systems. Rotation of the armature conductors in a constant magnetic field gives a generated voltage proportional to the armature speed.

DC motor applications

DC motors are very widely used. Examples include:

- Car auxiliaries – fans, wipers, windows, etc.
- Electrical vehicle drives.
- Industrial drives requiring speed and torque control.
- Large industrial drives: steel mills, mines.

Increasingly, however, these applications are being challenged by AC motors and brushless DC motors.

Brushless DC motors

A brushless DC motor uses electronic switching instead of a mechanical commutator and brushes. To simplify the construction, the motor is inverted: the field system is the rotating part, or *rotor*, and the armature coils are stationary. Figure 8-4 shows the components of a permanent-magnet brushless DC motor.



Figure 8-4: Brushless DC motor components.

Brushless DC motors are efficient, reliable and maintenance-free. Increasingly they are replacing conventional DC motors in many applications.

8.2 DC machine principles

The basic equations of the DC machine are related to the equations of the moving-coil transducer:

$$e = Blu \quad [2-1]$$

$$f = Bli \quad [2-2]$$

The electromagnetic force exerted on the armature conductors results in a *developed torque*, given by [4]:

$$T_d = K\phi_f i_a \quad [\text{Nm}] \quad (8-1)$$

In this equation, ϕ_f is the total flux of the field system, and i_a is the armature current. K is a constant for a particular machine, which depends on the number of turns and the method of interconnecting the armature coils.

The armature generated voltage, or EMF, is given by a similar equation:

$$e_a = K\phi_f \omega_r \quad [\text{V}] \quad (8-2)$$

where ω_r is the armature angular velocity. The armature constant K is the same as in equation 8-1 if all quantities are in SI units. In particular, the angular velocity must be in radians per second (rad/s) and not in rev/min. If the rotational speed is n_r rev/s or N_r rev/min, then:

$$\omega_r = 2\pi n_r = \frac{2\pi N_r}{60} \quad [\text{rad/s}] \quad (8-3)$$

Magnetisation characteristic

In a wound-field machine, the field flux ϕ_f depends on the field current i_f . From equation 8-2, if the speed ω_r is held constant, the flux is proportional to the armature generated voltage e_a . A graph of e_a against i_f is known as the *magnetisation characteristic* of the machine, and a typical curve is shown in figure 8-5 for a 3 kW motor.

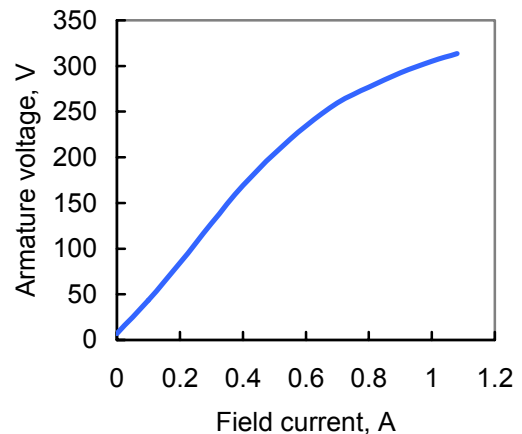


Figure 8-5: Magnetisation characteristic.

Up to the rated armature voltage of 220 V, when the field current is 0.5 A, the relationship between field current and armature voltage is almost linear. Note, however, that there is a small residual voltage when the field current is zero, due to residual magnetisation of the field system. If the field current is increased above 0.5 A, there is significant non-linearity, indicating saturation of part of the magnetic circuit.

Linear approximation

For the initial part of the magnetisation characteristic, it is a reasonable approximation to put $\phi_f \propto i_f$. Equations 8-1 and 8-2 then become:

$$T_d = K' i_f i_a \quad (8-4)$$

$$e_a = K' i_f \omega_r \quad (8-5)$$

These equations are commonly used for modelling DC machines in control systems. However, they are only valid below the knee of the magnetisation curve, and it is important not to use them outside this range.

Motor and generator operation

Figure 8-6 shows a symbolic representation of a DC machine, with the armature connected to a voltage source v_a .

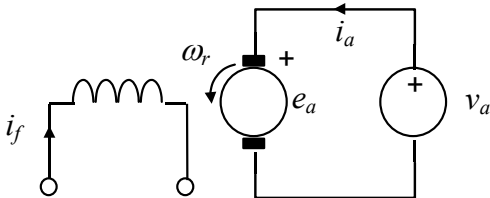


Figure 8-6: DC machine with voltage source.

If the armature has a resistance R_a , Kirchhoff's voltage law gives the armature voltage equation:

$$v_a = R_a i_a + e_a = R_a i_a + K \phi_f \omega_r \quad (8-6)$$

If current flows in the direction of the reference arrow for i_a , the source supplies power to the armature, so the machine is operating as a motor. If current flows in the opposite direction, the power flow is reversed, and the machine operates as a generator. The conditions are:

Motor operation: $e_a < v_a$ i_a is positive

Generator operation: $e_a > v_a$ i_a is negative

Since the generated voltage e_a depends on the rotational speed ω_r , it follows that a small increase in the speed can alter the direction of power flow, changing the operating mode from motoring to generating. This is an important consideration in applications such as electric vehicle drives.

Energy conversion

Multiplying equation 8-6 by i_a gives:

$$P_{in} = v_a i_a = R_a i_a^2 + K \phi_f \omega_r i_a \quad (8-7)$$

The last term can be expressed in terms of the developed torque from equation 8-1, giving:

$$P_{in} = R_a i_a^2 + T_d \omega_r \quad (8-8)$$

Thus the electrical input power to the armature is partly converted to heat in the armature resistance, and the rest is converted to mechanical form. The term $T_d \omega_r$ represents the gross mechanical power, but not all of this is available as output at the machine shaft. Some power is lost in bearing friction and air resistance, or *windage*, in the machine, so the useful output is $T \omega_r$, where T is the shaft torque.

Efficiency

The efficiency of a motor is defined in the usual way:

$$\text{Efficiency } \eta = \frac{\text{Mechanical output power}}{\text{Total electrical input power}} \quad (8-9)$$

The fraction of the input power lost as heat in the machine is therefore $1 - \eta$. Approximate efficiency and loss values are given in table 8-1 for wound-field DC motors of different sizes. Small permanent-magnet motors will be more efficient.

Table 8-1: DC motor efficiency and losses.

Power output	Efficiency	Losses
100 W	60%	40%
1 kW	80%	20%
10 kW	90%	10%
100 kW	95%	5%

8.3 Characteristics and control

Speed control

In a large motor, the total power loss is small. Since the armature power loss $R_a i_a^2$ is only a part of the total loss, this term must be small in comparison with the armature input power $v_a i_a$. It follows that:

$$R_a i_a \ll v_a \quad (8-10)$$

So $R_a i_a$ may be neglected in equation 8-6, giving:

$$v_a \approx K \phi_f \omega_r \quad (8-11)$$

The rotational speed is therefore:

$$\omega_r \approx \frac{v_a}{K \phi_f} \quad (8-12)$$

This equation shows that the speed is independent of the torque, provided inequality 8-10 holds.

Equation 8-12 is the basis of speed control. If the field flux ϕ_f is constant, the rotational speed is proportional to the applied voltage. The normal values of v_a and ϕ_f define the *base speed* ω_{r0} , and the speed can be reduced to zero by varying v_a .

Electronic controllers for DC motors deliver an adjustable voltage to the motor armature by phase-controlled rectification of the AC mains supply, using thyristors [4]. This is the basis of DC variable-speed drive systems, which are widely used in industry.

In a wound-field motor, it is possible to increase the speed above the base speed by reducing ϕ_f —known as *field weakening*. Only a limited speed increase is possible, for the following reason. The torque is related to the armature current through equation 8-1:

$$T_d = K \phi_f i_a \quad [8-1]$$

If ϕ_f is reduced, there will be a compensating increase in i_a to maintain the torque, and there is a risk of exceeding the current rating of the machine.

Small motors

In small motors, with power ratings below 1 kW, inequality 8-10 does not hold, so it is not possible to neglect the $R_a i_a$ term. The rotational speed then depends on the developed torque T_d , as may be seen by substituting for i_a in terms of T_d in equation 8-6:

$$\begin{aligned} v_a &= R_a i_a + K \phi_f \omega_r \\ &= \frac{R_a T_d}{K \phi_f} + K \phi_f \omega_r \end{aligned} \quad (8-13)$$

Thus, the speed is given by:

$$\omega_r = \frac{v_a}{K \phi_f} - \frac{R_a T_d}{(K \phi_f)^2} \quad (8-14)$$

A graph of speed against torque is a straight line, as shown in figure 8-7. The no-load speed, which is the speed when the torque is zero, is given by:

$$\omega_{r0} = \frac{v_a}{K \phi_f} \quad (8-15)$$

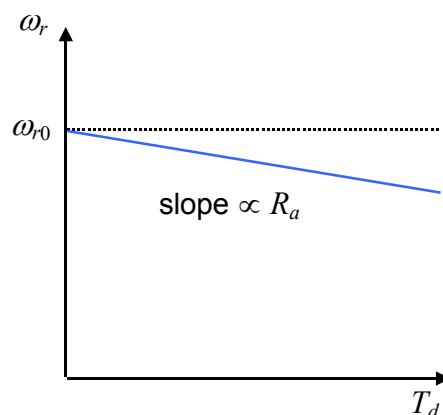


Figure 8-7: Speed-torque characteristic.

Worked example 8-1

A DC motor has an armature resistance of 1.0Ω , and a full-load rated armature current of 10 A . When the armature supply voltage is 200 V , the no-load speed is 400 rad/s . Determine the full-load speed.

Solution

From equation 8-15, $K\phi_f$ is given by:

$$K\phi_f = \frac{v_a}{\omega_{r0}} = \frac{200}{400} = 0.5$$

From equation 8-6, the full-load speed is:

$$\omega_r = \frac{v_a - R_a i_a}{K\phi_f} = \frac{200 - 1.0 \times 10}{0.5} = 380 \text{ rad/s}$$

Starting of DC motors

When the speed is zero, the armature-generated voltage is also zero. From equation 8-6, the corresponding armature current is given by:

$$i_{a0} = \frac{v_a}{R_a} \quad (8-16)$$

This is the *stalled armature current*, which is much larger than the normal running current. In a small motor, it is permissible to connect the armature directly to a constant-voltage supply. The armature can withstand the stalled current for a short time, and it will accelerate rapidly. As it does so, the generated voltage will rise and the current will fall to its normal value.

However, a large motor must not be started in this way because the armature resistance R_a is very low, and the stalled current would be large enough to cause serious damage. If an electronic controller is used for speed control, this is normally designed to limit the starting current to a safe value. If this form of control is not used, a variable resistance must be connected in series with the armature and the value progressively reduced to zero as the armature accelerates.

Series motors

So far, it has been assumed that the field flux ϕ_f is independent of the conditions in the armature. In a *series motor*, the field winding is designed to carry the full armature current, and it is connected in series with the armature as shown in figure 8-8.

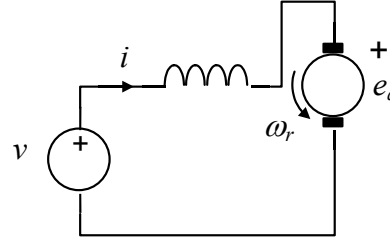


Figure 8-8: Series motor.

The characteristics of the series motor are readily deduced if the following assumptions are made:

- The field flux is proportional to the current.
- The resistance of the windings is negligible.

Equations 8-4 and 8-5 become:

$$T_d = K' i^2 \quad (8-17)$$

$$v = e_a = K' i \omega_r \quad (8-18)$$

Eliminating i gives the relationship between speed and torque:

$$\omega_r = \frac{v}{K' i} = \frac{v}{K' \sqrt{\frac{T_d}{K'}}} = \frac{v}{\sqrt{K' T_d}} \quad (8-19)$$

A graph plotted from equation 8-19 for a small motor is shown in figure 8-9.

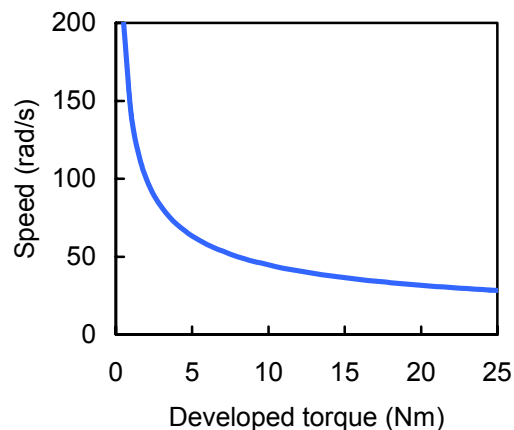


Figure 8-9: Speed/torque characteristic.

Two features of the graph should be noted:

- The motor can develop a very large torque at low speeds.
- When the developed torque is low, the speed is very high.

The high torque at low speeds is a useful feature, but the high speed associated with low torque can be dangerous.

Consider what happens if the external load torque is removed. The developed torque then just supplies the mechanical losses in the motor. In a small motor, the mechanical losses are usually high enough to limit the no-load speed to a safe value.

However, in motors with power ratings above about 1 kW, the losses are proportionately smaller, and the no-load speed would be dangerously high. Such motors must never be run without a load.

Applications of series DC motors include electric delivery vehicles, electric railways, and car starter motors.

Universal motors

If the polarity of the supply voltage v to a series motor is reversed, the direction of rotation will not change. This follows from the fact that the current reverses, thereby reversing the field flux. The torque direction therefore remains the same, so the motor will accelerate in the same direction as before. Since the flux is reversed, the generated voltage e_a is reversed to match the new direction of the applied voltage v .

A correctly designed series motor will therefore work on AC as well as DC. Such motors are termed *universal*. They are widely used in products such as portable power drills, and in domestic appliances such as food processors and vacuum cleaners.

Reversal of direction of rotation

To reverse the direction of rotation of a permanent-magnet motor, or a wound-field motor with a separate field supply, the polarity of the supply to the armature must be reversed.

With a series motor, reversing the polarity of the supply has no effect. To reverse this type of motor, the connections to the field or the armature must be reversed. Figure 8-10 shows the connections to the field reversed, so the direction of ω_r is reversed for the same polarity of e_a .

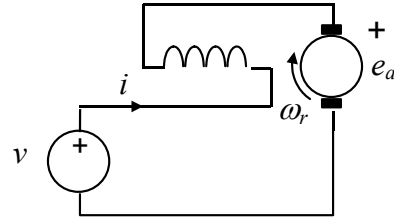


Figure 8-10: Reversal of a series motor.

8.4 Effect of size

To examine the effect of size on the performance of a DC motor, consider three motors:

- Motor 1 is the reference design.
- Motor 2 is identical in design to motor 1 in all respects, except that each of its dimensions is exactly double the corresponding dimension in motor 1. The ratio of armature current to field current is the same as in motor 1.
- Motor 3 is the same size as motor 2, but the armature current has been increased to give the same heat loss conditions as in motor 1.

All coils have the same numbers of turns in the three machines, the armature constant K is the same, and the magnetic flux density at corresponding points is the same.

If lengths are multiplied by 2, then areas are multiplied by $2^2 = 4$ and volumes are multiplied by $2^3 = 8$. Table 8-2 shows the effect on various quantities, where subscripts 1, 2 and 3 refer to motors 1, 2 and 3 respectively.

The power loss per unit area determines the temperature rise of the armature, so motor 2 will run at a lower temperature than motor 1. It is therefore possible to increase the armature current,

which has been done in motor 3 to make the power loss per unit area the same as in motor 1.

The armature efficiency of a motor may be defined as the ratio of armature output power to electrical input power. Since the input is the sum of the output and the power loss, we have the following expression for the armature efficiency:

$$\eta_a = \frac{P_{out}}{P_{in}} = \frac{P_a}{P_a + P_l} = \frac{1}{1 + \frac{P_l}{P_a}} = \frac{1}{1 + \zeta} \quad (8-20)$$

where ζ is the ratio of power loss to power output.

We have the following comparisons between motor 3 and motor 1:

- The two motors run at the same temperature.
- The motor volume has increased by a factor of 8, but the power output has increased by a factor of $8\sqrt{2} = 11.3$.
- The power loss ratio has fallen by a factor of $2\sqrt{2} = 2.83$, so the efficiency has improved.

Similar comparisons apply to other electromagnetic devices such as transformers and AC motors. The rule in the electromagnetic world is “the bigger the better”.

Table 8-2: Effect of size on motor performance.

Quantity	Equation	Motor 2	Motor 3
Field flux	$\phi_f = BA_f$	$\phi_{f2} = 4\phi_{f1}$	$\phi_{f3} = 4\phi_{f1}$
Armature generated voltage	$e_a = K\phi_f\omega_r$	$e_{a2} = 4e_{a1}$	$e_{a3} = 4e_{a1}$
Field current	$N_f i_f = Hl_f$	$i_{f2} = 2i_{f1}$	$i_{f3} = 2i_{f1}$
Armature current	$i_a = K_i i_f$	$i_{a2} = 2i_{a1}$	$i_{a3} = 2\sqrt{2}i_{a1}$
Armature resistance	$R_a = \frac{\rho l_e}{A_e}$	$R_{a2} = \frac{R_{a1}}{2}$	$R_{a3} = \frac{R_{a1}}{2}$
Armature output power	$P_a = e_a i_a$	$P_{a2} = 8P_{a1}$	$P_{a3} = 8\sqrt{2}P_{a1}$
Armature power loss	$P_l = i_a^2 R_a$	$P_{l2} = 2P_{l1}$	$P_{l3} = 4P_{l1}$
Armature power loss per unit surface area	$S = \frac{P_l}{A_s}$	$S_2 = \frac{S_1}{2}$	$S_3 = S_1$
Power loss ratio	$\zeta = \frac{P_l}{P_a}$	$\zeta_2 = \frac{\zeta_1}{4}$	$\zeta_3 = \frac{\zeta_1}{2\sqrt{2}}$

9 VARIABLE RELUCTANCE DEVICES

9.1 Force and inductance

Consider again the electromagnet of section 4.6, but suppose that the armature moves upward towards the poles with velocity u , as shown in Figure 9-1. It is assumed that the current i is held constant by an external source.

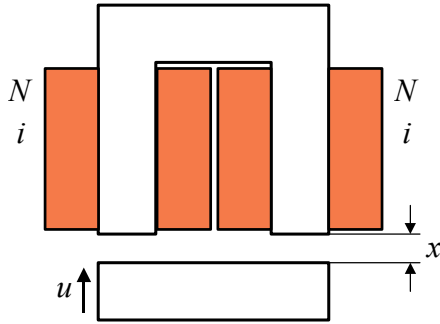


Figure 9-1: Electromagnet with motion.

The airgap length will be a variable, denoted by x , so the velocity is given by:

$$u = \frac{dx}{dt} \quad (9-1)$$

If the inductance of the coil is L , the induced voltage is given by:

$$\begin{aligned} e &= \frac{d}{dt}(Li) = L \frac{di}{dt} + i \frac{dL}{dt} = i \frac{dL}{dt} \\ &= i \frac{dL}{dx} \cdot \frac{dx}{dt} = iu \frac{dL}{dx} \end{aligned} \quad (9-2)$$

Note that the current is held constant, so $di/dt = 0$. The electrical power input from the source is:

$$P_e = ei = i^2 u \frac{dL}{dx} \quad (9-3)$$

If the upward force on the armature is f , then the mechanical power output is:

$$P_m = fu \quad (9-4)$$

The stored energy in the magnetic field is given by:

$$W_m = \frac{1}{2} Li^2 \quad (9-5)$$

Since energy is conserved, we have:

$$P_e = P_m + \frac{dW_m}{dt} \quad (9-6)$$

Substituting for P_e , P_m and W_m gives:

$$i^2 u \frac{dL}{dx} = fu + \frac{d}{dt} \left(\frac{1}{2} Li^2 \right) = fu + \frac{1}{2} i^2 u \frac{dL}{dx} \quad (9-7)$$

Thus:

$$fu = \frac{1}{2} i^2 u \frac{dL}{dx} \quad (9-8)$$

The force is therefore given by:

$$f = \frac{1}{2} i^2 \frac{dL}{dx} \quad (9-9)$$

In general, if the displacement of any part of a system causes a change in the inductance, then the x -component of force on that part is:

$$f_x = \frac{1}{2} i^2 \frac{\partial L}{\partial x} \quad (9-10)$$

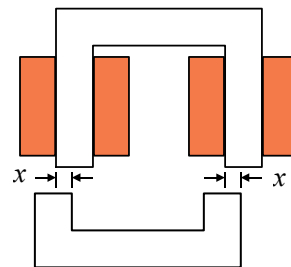
Similarly, if rotation through an angle θ causes a change in the inductance, then the corresponding torque is:

$$T_\theta = \frac{1}{2} i^2 \frac{\partial L}{\partial \theta} \quad (9-11)$$

Equations 9-10 and 9-11 can be very useful for determining forces and torques in devices where it is possible to calculate or measure the inductance as a function of position.

Worked example 9-1

The figure below shows an electromagnet with a shaped armature displaced from its position of alignment with the poles. The depth w perpendicular to the diagram is 25 mm, the number of turns N on each coil is 600, the current i is 2.0 A, and the flux density B_g in the airgap is 0.503 T. If x is the length of the overlapping portion of each pole and the armature, determine the inductance as a function of x , and hence calculate the alignment force. Assume that the field in the airgap is confined to the overlapping region.



Solution

The flux in the overlapping region is:

$$\phi = B_g A = B_g wx$$

This flux passes through each coil, so the inductance is:

$$L = \frac{\lambda}{i} = \frac{2N\phi}{i} = \frac{2NB_g wx}{i}$$

The alignment force is:

$$f_x = \frac{1}{2} i^2 \frac{\partial L}{\partial x} = \frac{1}{2} i^2 \frac{\partial}{\partial x} \left(\frac{2NB_g wx}{i} \right) = NiB_g w$$

$$= 600 \times 2.0 \times 0.503 \times 25 \times 10^{-3} = 15.1 \text{ N}$$

Note that this result is independent of the value of x .

9.2 Force and reluctance

Referring to Figure 9-1, let $N_t = 2N$ be the total number of turns on the electromagnet, and let ϕ be the magnetic flux, which is assumed constant round the magnetic circuit. The inductance is:

$$L = \frac{\lambda}{i} = \frac{N_t \phi}{i} \quad (9-12)$$

For the magnetic circuit:

$$F = N_t i = R \phi \quad (9-13)$$

Thus, the flux is given by:

$$\phi = \frac{F}{R} = \frac{N_t i}{R} \quad (9-14)$$

Substituting in equation 9-12 gives:

$$L = \frac{N_t \phi}{i} = \frac{N_t \left(\frac{N_t i}{R} \right)}{i} = \frac{N_t^2}{R} \quad (9-15)$$

From equation 9-13, the current is:

$$i = \frac{R \phi}{N_t} \quad (9-16)$$

Substituting for L and i in equation 9-10 gives:

$$f_x = \frac{1}{2} i^2 \frac{\partial L}{\partial x} = \frac{1}{2} \left(\frac{R \phi}{N_t} \right)^2 \frac{\partial}{\partial x} \left(\frac{N_t^2}{R} \right)$$

$$= -\frac{1}{2} \phi^2 \frac{\partial R}{\partial x} \quad (9-17)$$

There is a similar result for torque:

$$T_\theta = -\frac{1}{2} \phi^2 \frac{\partial R}{\partial \theta} \quad (9-18)$$

Electromagnet lift force

For the electromagnet in Figure 9-1, if A_p is the area of each pole face and B_g is the airgap flux density, then:

$$R = \frac{l}{\mu_0 A} = \frac{2x}{\mu_0 A_p} \quad (9-19)$$

$$\phi = B_g A_p \quad (9-20)$$

From equation 9-17, the lift force is:

$$f_x = -\frac{1}{2} \phi^2 \frac{\partial R}{\partial x}$$

$$= -\frac{1}{2} (B_g A_p)^2 \cdot \frac{2}{\mu_0 A_p} = -\frac{B_g^2 A_p}{\mu_0} \quad (9-21)$$

Thus the lift force per unit area is:

$$\frac{f}{A} = \frac{|f_x|}{2A_p} = \frac{1}{2} \frac{B_g^2}{\mu_0} \quad (9-22)$$

This is a derivation of equation 2-12, which was used to calculate the force of attraction in electromagnets. Since the result is expressed in terms of the airgap flux density, it is also applicable to other devices such as permanent magnets. See reference [4] for an alternative treatment.

9.3 Variable reluctance devices

Equations 9-17 and 9-18 are the basis of a variety of practical devices that develop a force or a torque because the reluctance of the magnetic circuit can change. A good example is the variable-reluctance *stepper motor* [4], where the rotor moves through a defined angle when currents are switched from one stator coil to another. Figure 9-2 shows a 3D model of a stepper motor with a 4-pole rotor and a 6-pole stator.



Figure 9-2: Stepper motor model.

Figure 9-3 shows a cross-section of the model. Coils on opposite pairs of poles are connected in series to form one phase, so this is a 3-phase motor.

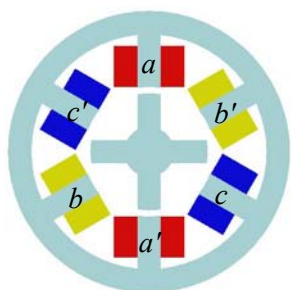


Figure 9-3: Model cross-section.

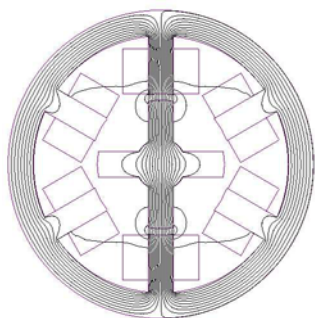


Figure 9-4: Flux plot: phase a energised.

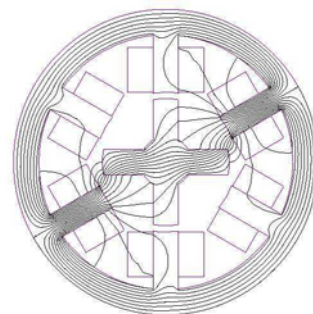


Figure 9-5: Flux plot: phase b energised.

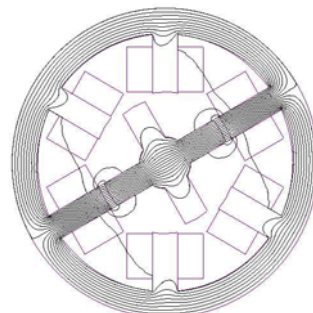


Figure 9-6: Flux plot: new rotor position.

Figure 9-4 shows the flux plot when phase *a* is energised, so that the rotor is held in alignment with phase *a*. If the rotor remains in this position, but current is switched from phase *a* to phase *b*, the resulting flux plot is shown in figure 9-5. Magnetic forces act on the misaligned poles, and if the rotor is free to move it will take up a new position of alignment with phase *b* as shown in figure 9-6. Note that the field pattern has rotated 60° clockwise, but the rotor has moved 30° counter-clockwise. There is a similar effect when current is switched from phase *b* to phase *c*, and then from phase *c* to phase *a*. Each switching transition causes the rotor to turn through the *step angle* of 30° . Much smaller step angles are possible with the *hybrid stepper motor* [4], which employs a permanent magnet in an ingenious way to enhance the variable-reluctance alignment effect. A typical step angle with this type of motor is 1.8° , giving 200 steps per revolution.

Stepper motors are essentially position control devices, where it is required to turn the shaft through a specified angle. The electronic controller for the motor generates the required number of switching actions for the coil currents. Continuous rotation is also possible, and variable-reluctance motors designed to operate in this way are known as *switched reluctance motors*. In some applications, switched reluctance motors are alternatives to the widely used *induction motors* considered in section 10. Figure 9-7 shows the stator, rotor and drive electronics of a typical switched reluctance motor.



Figure 9-7: Switched reluctance motor.
(Switched Reluctance Drives Ltd)

10 AC MACHINES

10.1 Introduction to AC machines

Review of 3-phase systems

Industrial AC motors use 3-phase alternating current to generate a rotating magnetic field from stationary windings. The 3-phase supply may be taken from the AC mains, or it may be generated electronically with an inverter. In either case, the requirement is a symmetrical set of sinusoidal currents with relative phase displacements of 120° .

Formally, 3-phase sets of currents and voltages may be defined as follows in the time domain:

$$\begin{aligned} i_a &= I_m \cos(\omega t) \\ i_b &= I_m \cos(\omega t - 120^\circ) \\ i_c &= I_m \cos(\omega t - 240^\circ) \end{aligned} \quad (10-1)$$

$$\begin{aligned} v_a &= V_m \cos(\omega t + \phi) \\ v_b &= V_m \cos(\omega t + \phi - 120^\circ) \\ v_c &= V_m \cos(\omega t + \phi - 240^\circ) \end{aligned} \quad (10-2)$$

These quantities may be represented in the frequency domain by phasors in the usual way.

Figure 10-1 shows the star connection of a 3-phase AC source to three lines, and figure 10-2 shows the corresponding voltage phasor diagram.

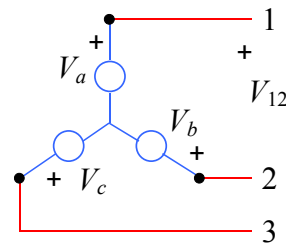


Figure 10-1: 3-phase star connection.

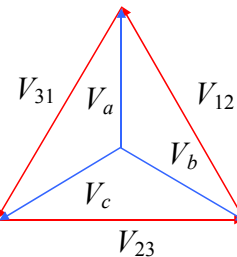


Figure 10-2: Star connection: voltage phasors.

Figure 10-3 shows the delta connection of a 3-phase AC source to three lines, and figure 10-4 shows the corresponding current phasor diagram.

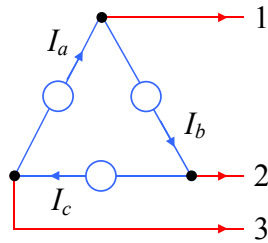


Figure 10-3: 3-phase delta connection.

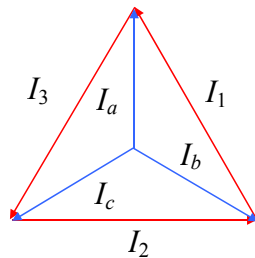


Figure 10-4: Delta connection: current phasors.

From these diagrams, we have the following relationships between the magnitudes of line and phase quantities:

Star	Delta
$V_{line} = \sqrt{3}V_{phase}$	$V_{line} = V_{phase}$
$I_{line} = I_{phase}$	$I_{line} = \sqrt{3}I_{phase}$

Rotating magnetic field

In AC motors, the stationary part is termed the *stator*, and the rotating part is the *rotor*. Figure 10-5 shows the stator of a small AC motor with a 3-phase winding.

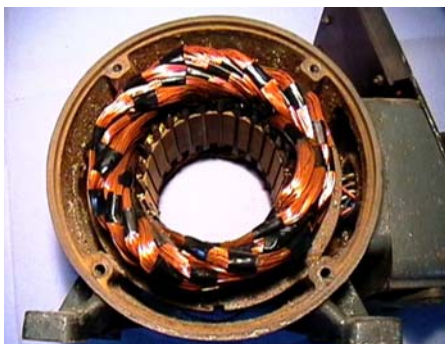


Figure 10-5: 3-phase AC motor winding.

Coils are arranged in slots in a laminated steel core, rather like the armature coils in a DC motor. The function of the winding is to route currents to slots from the three phases, as shown in the simplified diagram of figure 10-6. This diagram also shows a steel cylinder in the centre, representing the rotor of the motor.

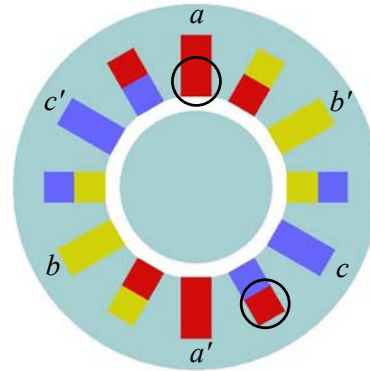


Figure 10-6: 3-phase conductor groups.

The groups labelled *a*, *b*, *c* carry currents in the positive direction, and the groups labelled *a'*, *b'*, *c'* carry currents in the negative direction.

All coils have the same shape, with one coil side at the bottom of a slot and the other side at the top of a slot, as shown by the pair of circles in figure 10-6.

If phase *a* is energised on its own with positive current, the resulting magnetic field pattern is shown in figure 10-7. The corresponding patterns for phases *b* and *c* are shown in figures 10-8 and 10-9.

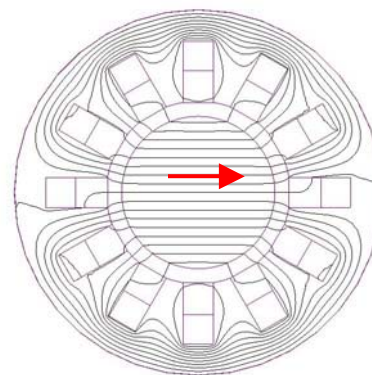


Figure 10-7: Flux plot for phase a.

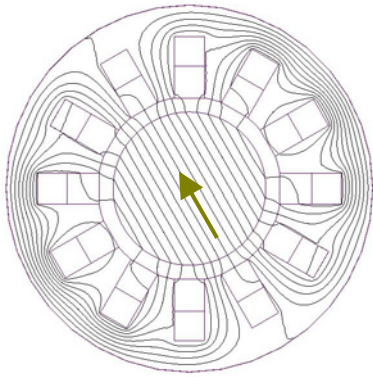


Figure 10-8: Flux plot for phase *b*.

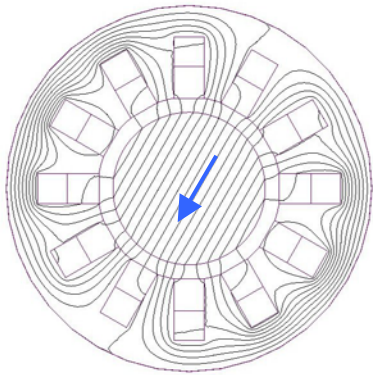


Figure 10-9: Flux plot for phase *c*.

When all three phases are energised, the individual phase fields combine to give a resultant field. Consider the instant when $\omega t = 0$. From equation 10-1, the currents in the three phases are:

$$i_a = +I_m, \quad i_b = -\frac{1}{2}I_m, \quad i_c = -\frac{1}{2}I_m$$

Since the currents in phases *b* and *c* are negative, the component fields are in the directions shown in figure 10-10. They combine to give a resultant in the same direction as for phase *a*, so a flux plot of the resultant field is the same as for phase *a* alone. As time advances, the currents in each phase change, so the resultant field pattern changes. Figures 10-11, 10-12 and 10-13 show the patterns when $\omega t = 30^\circ$, 60° and 90° respectively.

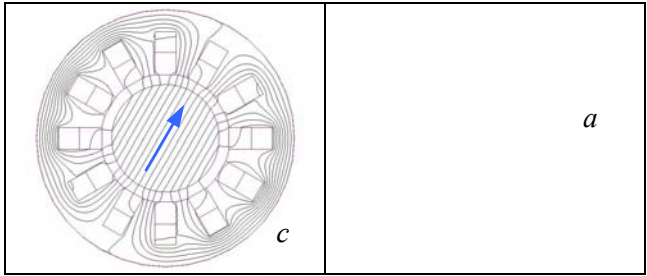
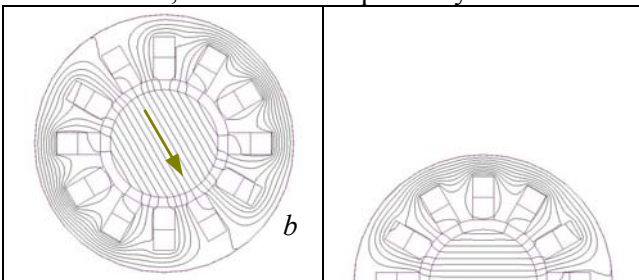


Figure 10-10: Component fields when $\omega t = 0$.

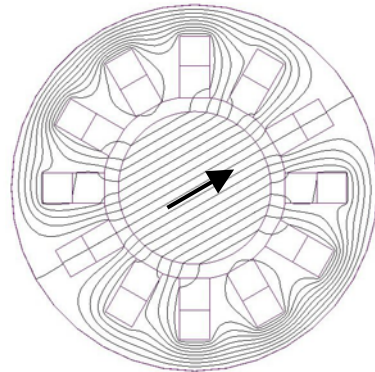


Figure 10-11: Resultant field when $\omega t = 30^\circ$.

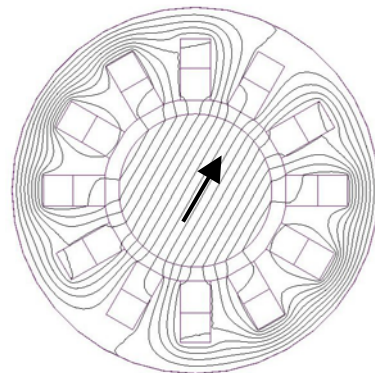


Figure 10-12: Resultant field when $\omega t = 60^\circ$.

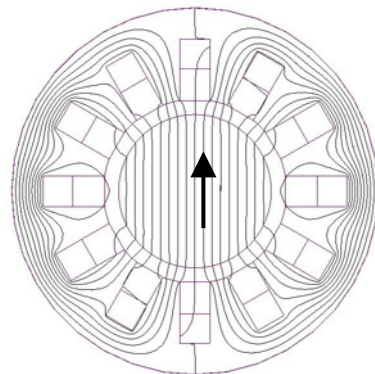


Figure 10-13: Resultant field when $\omega t = 90^\circ$.

Notice that the magnetic field pattern has rotated by the same angle as the time phase of the currents. The result is a rotating magnetic field, which makes

10.2 Synchronous motors

In a synchronous motor, the rotor is magnetised and it runs at the same speed as the rotating magnetic field. Permanent-magnet rotors are common in small machines, so the machine structure is similar to that of the brushless DC motor shown in figure 8-4. Figure 10-16 is a simplified model of the structure of this type of motor, where the rotor has surface-mounted blocks of permanent-magnet material.

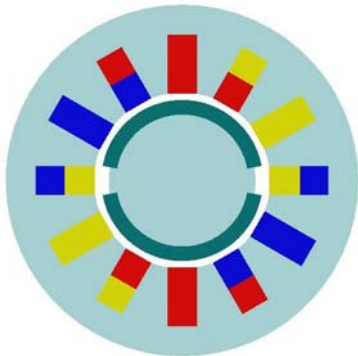


Figure 10-16: 2-pole PM synchronous motor.

In large sizes, synchronous motors have a field winding on the rotor instead of permanent magnets.

The synchronous motor develops a torque when the magnetic axis of the rotor is displaced from the axis of the stator magnetic field. Figure 10-17 shows the resulting field pattern for the motor of figure 10-16 when the displacement angle is 90° , which gives the maximum torque for a given current. The current has been chosen so that the axis of the resultant field is at approximately 45° to the magnetic axis of the rotor, which is vertical in figure 10-16.

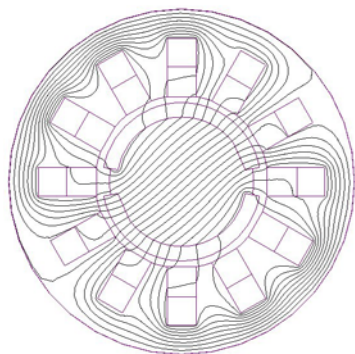


Figure 10-17: Synchronous motor flux plot.

Torque characteristic

The synchronous motor torque characteristic is usually expressed in terms of the *load angle* δ , which is the angle between the axis of the resultant magnetic field and the magnetic axis of the rotor. It may be shown [4] that an approximate expression for the torque is:

$$T_d = \frac{3VE\sin\delta}{X_s} \quad [\text{Nm}] \quad (10-6)$$

In this equation, V is the terminal voltage per phase, E is the voltage induced by the magnetised rotor, and X_s is the effective reactance of the stator winding. The torque has a maximum value $T_{d\max}$ when $\delta = 90^\circ$. Figure 10-18 shows a graph of the normalised torque, $T_d / T_{d\max}$, against δ .

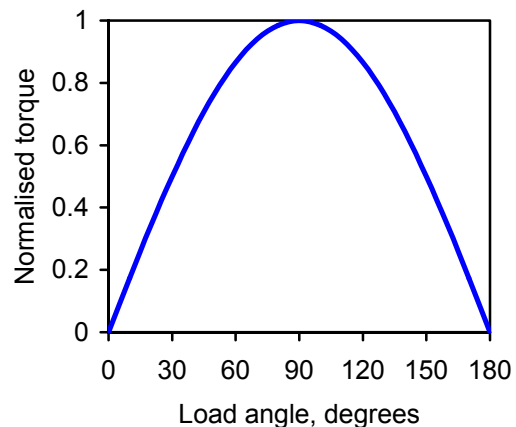


Figure 10-18: Synchronous motor torque.

If the mechanical load on the motor exceeds $T_{d\max}$, the rotor will be pulled out of synchronism with the rotating field, and the motor will stall. $T_{d\max}$ is therefore known as the *pullout torque*.

Synchronous machine applications

As with DC machines, the direction of power flow in a synchronous machine can be reversed so that it functions as a generator. Since the speed cannot change, the torque must reverse, and the load angle will change sign, indicating that the magnetic axis of the rotor is ahead of the axis of the stator magnetic field. Figure 10-19 shows the torque characteristic for positive and negative load angles.

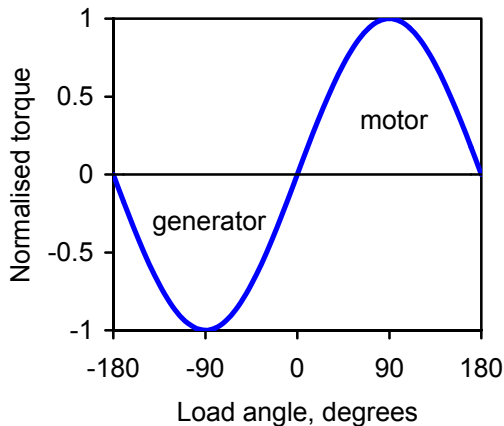


Figure 10-19: Synchronous machine torque.

Nearly all the AC generators in power stations are synchronous machines. Car alternators are also synchronous generators, with a built-in bridge rectifier to give a DC output.

Large synchronous motors are used in industry as high-efficiency constant-speed drives. The electrical characteristics can be controlled by varying the rotor field current, so that the motor can be used for power-factor correction [4].

Small permanent-magnet synchronous motors, supplied from variable-frequency electronic inverters, are used as industrial-variable speed drives. This topic is discussed further in section 10.3 in the context of induction motors.

The torque characteristic shown in figure 10-19 has an important practical consequence. If the rotor loses synchronism with the rotating field, the load angle will change continuously. The torque will be alternately positive and negative, with a mean value of zero. Synchronous motors are therefore not inherently self-starting. Induction machines, considered in section 10.3, do not have this limitation, and the induction principle is generally used for starting synchronous motors.

Linear synchronous motors

Many applications require motion in a straight line rather than rotary motion, for example in automation systems. Linear motors provide this. In concept, a linear motor is a rotary motor opened out flat. Figure 10-20 shows the linear counterpart of the rotating field model of figure 10-6, and figure 10-21(a)-(d) show the field patterns when $\omega t = 0^\circ$, 30° , 60° and 90° respectively.

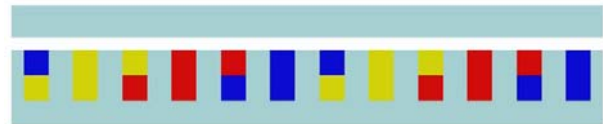
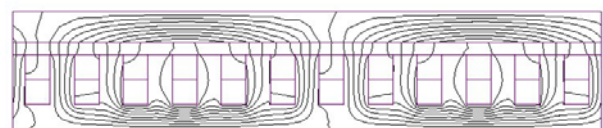
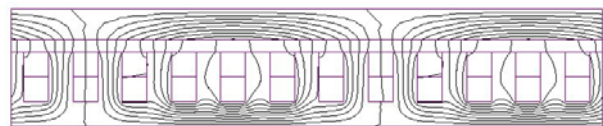


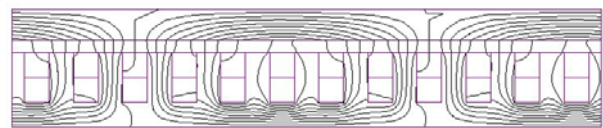
Figure 10-20: Travelling field model.



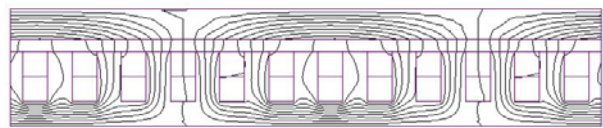
(a)



(b)



(c)



(d)

Figure 10-21: Flux plots for travelling field: (a) 0° , (b) 30° , (c) 60° , (d) 90°

Figure 10-22 shows a model of a linear synchronous motor derived from the rotary model of figure 10-16, and figure 10-23 shows the corresponding flux plot. The lower part with the windings is termed the *primary*, and the upper part with the permanent magnets is termed the *secondary*. Figure 10-24 shows the primary winding of a small linear motor.

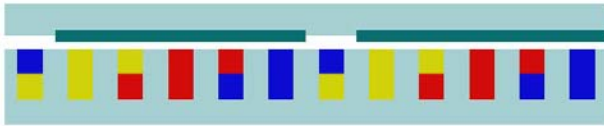


Figure 10-22: Linear synchronous motor.

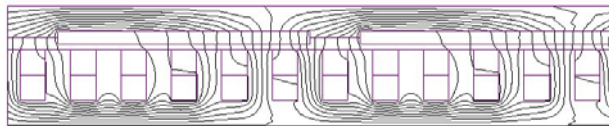


Figure 10-23: Linear motor flux plot.



Figure 10-24: Linear motor primary.

The German Transrapid advanced passenger vehicle shown in figure 2-7 uses linear synchronous motors for propulsion, with the primary on the track.

An alternative form of linear synchronous motor is the tubular motor, which is effectively a flat motor rolled up into a tube. Figure 10-25 shows a commercial motor of this kind. The primary winding comprises simple circular coils surrounded by a heat sink for cooling, and the secondary magnets are enclosed in a tube for protection.

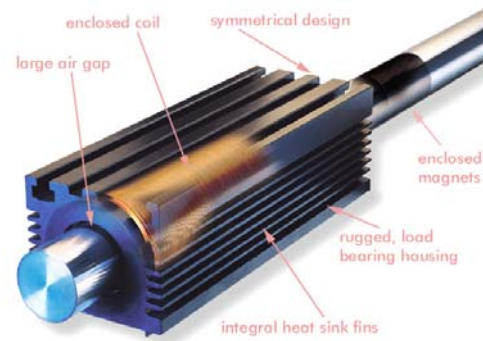


Figure 10-25: Tubular linear motor. (Copley Motion Systems)

10.3 Induction motors

Induction motors exploit the rotating magnetic field in quite a different way from synchronous motors. Instead of a magnetised rotor, an induction motor has a rotor with short-circuited conductors. The motion of the rotating field induces currents in the conductors, which in turn interact with the field to develop a torque on the rotor.

Figure 10-26 shows a typical rotor for a small induction motor, where conductors in the form of aluminium bars are placed in slots in the laminated steel core. At each end, the bars are connected to short-circuiting aluminium rings known as end-rings. In this kind of rotor, the bars and end-rings are made in a single operation by die-casting.



Figure 10-26: Induction motor rotor.

To develop a torque, the rotor of an induction motor must run more slowly than the rotating magnetic field. Unlike synchronous motors, induction motors are inherently self-starting. The rotor will accelerate from rest until it is running just below the *synchronous speed*, which is the speed of the rotating field.

Induction motors are simple and robust, and their self-starting capability is a particular

advantage. About 90 per cent of industrial drives are induction motors. A typical small motor is shown in figure 10-27.



Figure 10-27: Small induction motor.
(ABB Automation)

Induction motor characteristics

An essential feature of induction motors is the speed difference between the rotor and the rotating magnetic field, which is known as *slip*. There must be some slip for currents to be induced in the rotor conductors, and the current magnitude increases with the slip. It follows that the developed torque varies with the slip, and therefore with the rotor speed. Figure 10-28 shows a typical torque/speed characteristic for a 4-pole 3 kW induction motor, operated from a constant-voltage 50 Hz supply.

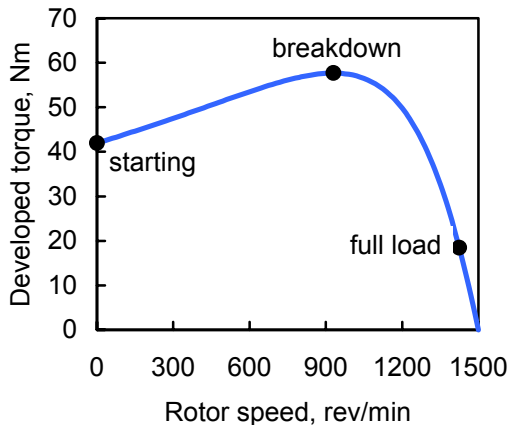


Figure 10-28: Torque/speed characteristic.

The maximum torque is known as the *breakdown torque*. If a mechanical load torque greater than this is applied to the motor, it will stall. Also shown in figure 10-28 is the full-load operating point, and the starting torque when the speed is zero. The

torque is zero at the synchronous speed of 1500 rev/min.

Another important motor characteristic is the input current. Since the rotor current increases with slip, and therefore with decreasing rotor speed, there must be a corresponding change in the stator current, as shown in figure 10-29.

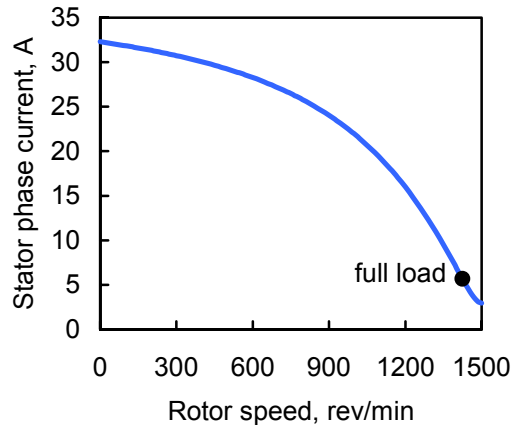


Figure 10-29: Current/speed characteristic.

Two points should be noticed about the current:

- At the synchronous speed of 1500 rev/min, although the torque is zero, the current is about half the full-load value. This is the *magnetising current*, which creates the magnetic flux in the motor.
- When the speed is zero, the starting current is about six times the full-load current.

Slip, power, and efficiency

It is useful to define a quantity *s*, known as the *fractional slip*, as follows:

$$s = \frac{\text{slip speed}}{\text{synchronous speed}} = \frac{N_s - N_r}{N_s} = \frac{\omega_s - \omega_r}{\omega_s} \quad (10-7)$$

From equation 10-7, the slip speed and the rotor speed in terms of *s* are given by:

$$\omega_s - \omega_r = s\omega_s \quad (10-8)$$

$$\omega_r = \omega_s - s\omega_s = (1 - s)\omega_s \quad (10-9)$$

The rotating magnetic field exerts a torque T_d on the rotor, and does work at the rate $\omega_s T_d$. This represents an input of power to the rotor. The rotor revolves at a speed ω_r , and therefore does work at the rate $\omega_r T_d$, which represents the mechanical output power of the rotor. The difference between these two powers represents power lost in the resistance of the rotor. As in the DC motor, there will be mechanical losses in the motor, so the shaft torque T is less than T_d . We have the following rotor power relationships:

$$\text{Rotor input: } P_{em} = \omega_s T_d \quad (10-10)$$

$$\text{Rotor output: } P_{out} = \omega_r T_d = (1-s)\omega_s T_d \quad (10-11)$$

$$\text{Rotor loss: } P_{loss} = (\omega_s - \omega_r)T_d = s\omega_s T_d \quad (10-12)$$

Thus, a fraction $(1-s)$ of the rotor input power is converted into mechanical power, and a fraction s is lost as heat in the rotor conductors. The quantity $(1-s)$ is termed the *rotor efficiency*. Since there are other losses in the motor, the overall efficiency will be less than the rotor efficiency.

For high efficiency, the fractional slip s should be as small as possible. In large motors, with power ratings of 100 kW or more, the value of s at full load is about 2%. For small motors, with power ratings below about 10 kW, the corresponding value is about 5%. It follows that the full-load speed of an induction motor is never more than a few per cent less than the synchronous speed.

Induction motor model

When the rotor is stationary, the induction motor behaves as a 3-phase transformer with a short-circuited secondary. The equivalent-circuit model of the transformer in figure 6-10 applies to this condition. When the rotor moves, the voltage induced in the rotor will depend on the relative motion. It may be shown [4] that this can be represented by a simple change to the equivalent circuit: the secondary resistance is not constant, but depends on the fractional slip s . If core losses are neglected, the circuit for one phase takes the form shown in figure 10-30.

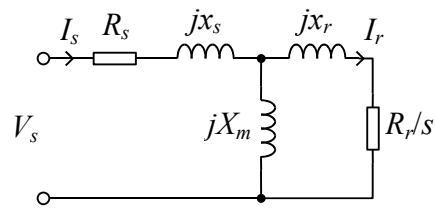


Figure 10-30: Induction motor equivalent circuit.

The developed torque is obtained by equating the power absorbed in the resistance R_r/s to the rotor input power from equation 10-10, giving the result:

$$T_d = \frac{3}{\omega_s} |I_r|^2 \frac{R_r}{s} \quad [\text{Nm}] \quad (10-13)$$

The graphs in figures 10-28 and 10-29 are derived from this model, for a motor with the following parameter values per phase:

$$R_s = 2.27 \, \Omega, \quad R_r = 2.28 \, \Omega, \quad x_s = x_r = 2.83 \, \Omega, \\ X_m = 74.8 \, \Omega.$$

Motor driving a load

When a motor drives a mechanical load, the operating point is given by the intersection of the motor torque/speed characteristic with the corresponding load characteristic, as shown in figure 10-31.

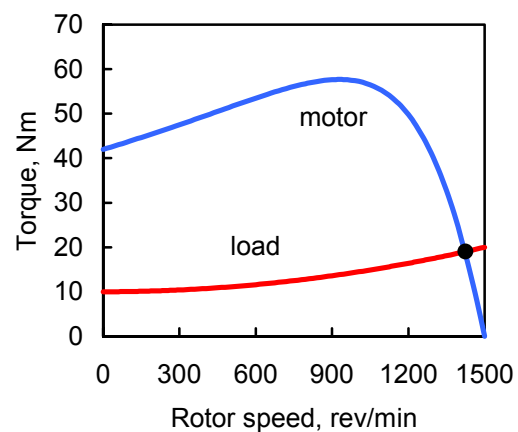


Figure 10-31: Motor driving a load.

Speed control

In normal operation, the speed of an induction motor is close to the synchronous speed, which in turn depends on the frequency of the AC supply. To vary the speed, the motor is normally supplied from a variable-frequency inverter [4], although other methods of speed control are possible [4]. The inverter also varies the voltage in proportion to the frequency, to maintain a constant magnetic flux density in the motor. For an explanation, see the discussion of voltage and frequency relationships for the transformer in section 6.2; similar considerations apply to the induction motor.

Figure 10-32 shows a set of motor torque-speed curves for different frequencies, together with the mechanical load curve. At each frequency, the operating point is the intersection of the load curve with the motor curve.

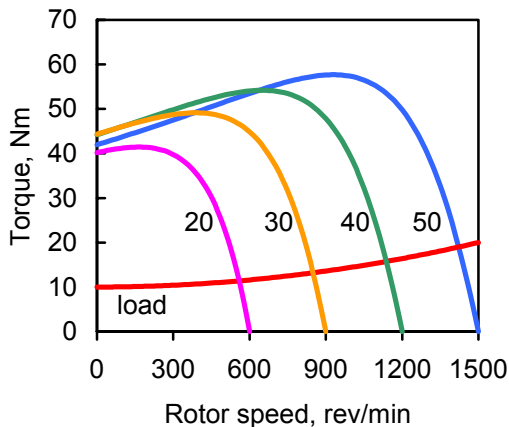


Figure 10-32: Variable-frequency operation: 20, 30, 40 and 50 Hz.

The combination of an induction motor and a variable-frequency inverter makes an excellent general-purpose variable-speed drive. This has displaced the voltage-controlled DC motor from its dominant position. An inverter is much more complex than a voltage controller, but inverter costs have fallen, and an induction motor is much cheaper than a DC motor of the same power rating. Induction motors also have the advantage of low maintenance and high reliability.

Single-phase induction motors

For domestic and light industrial applications, induction motors need to operate from a single-phase supply. Instead of a 3-phase winding, many single-phase motors have a 2-phase winding and a capacitor to give a phase shift of approximately 90° between the currents, as shown in figure 10-33 [4].

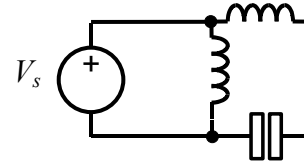


Figure 10-33: Single-phase induction motor.

The phases of a 2-phase winding are displaced in space by 90° (for a 2-pole machine), corresponding to the 90° time-phase displacement between the currents. As with a 3-phase winding, it may be shown [4] that a 2-phase winding with 2-phase currents can produce a rotating magnetic field.

Once the motor has started, it is possible to disconnect the capacitor, and the motor will continue to run with a single winding, but the performance is not as good.

The shaded-pole principle [4] is used in very small single-phase induction motors such as the one shown in figure 10-34. They are widely used for driving small cooling fans.

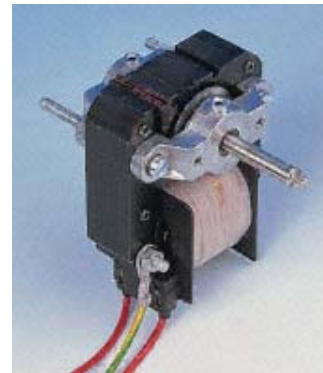


Figure 10-34: Shaded-pole induction motor. (RS Components Ltd)

This type of motor has a single winding on the stator, energising a pair of poles. A part of each pole is enclosed by a ring of copper, known as a *shading coil*. Currents are induced in the rings, giving the effect of a rudimentary 2-phase winding.

Linear induction motors

As with synchronous motors, a linear form of the induction motor can be derived by ‘unrolling’ a rotary motor, so the travelling-field model of figure 10-20 is applicable. A model of a simple form of linear induction motor is shown in figure 10-35.

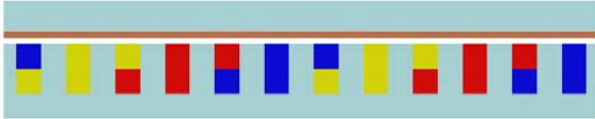


Figure 10-35: Linear induction motor model.

The primary with a 3-phase winding is similar to the primary of the linear synchronous motor in figure 10-22, and the secondary comprises a conducting plate on a steel backing. The plate, which is normally made from a material such as copper or aluminium, takes the place of the rotor conductors, and the steel backing provides the flux return path.

Figure 10-36 shows the flux plot for this model when $\omega t = 0$, and figure 10-37 shows a colour map of the currents in the primary winding and the secondary conducting plate at this instant of time. The colour range from blue to red corresponds to the range from minimum to maximum current.

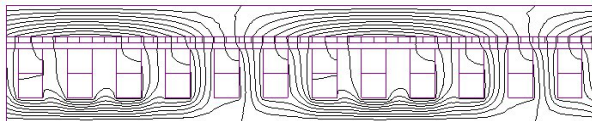


Figure 10-36: Flux plot for $\omega t = 0$.

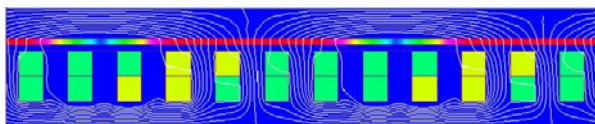


Figure 10-37: Current magnitude for $\omega t = 0$.

Like rotary induction motors, linear induction motors are robust and self-starting. They have been applied in airport baggage handling systems and theme park rides, and in advanced transport systems such as the magnetically levitated vehicle shown in figure 10-37.



Figure 10-38: Maglev vehicle with linear induction motor propulsion. (HSST Corporation)

The behaviour of a linear induction motor can differ significantly from that of an equivalent rotary motor [4]. Linear motors usually have large airgaps, so the magnetising current is larger than in a rotary motor. In addition, a linear motor has two ends that have no counterpart in a rotary motor. Transient currents can be generated as secondary conducting material enters the active region at one end and leaves it at the other end. These transient currents can reduce the force on the secondary and increase the losses, particularly at high speeds.

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12 APPENDICES

12.1 Silicon steel characteristics

The magnetisation characteristics in figures 12-1 and 12-2, are extended and expanded versions of figure 3-18. They are derived from the characteristics for Carpenter silicon steel from the material library of the electromagnetic simulation software MagNet (Infolytica Corporation). Numerical B and H values are as follows.

H (kA/m)	B (T)
0	0
0.05	0.431
0.07	0.66
0.1	0.845
0.14	0.984
0.2	1.101
0.3	1.214
0.4	1.286
0.5	1.335
0.7	1.4
1	1.453
1.4	1.485
2	1.522
3	1.58
4	1.62
5	1.652
7	1.708
10	1.773
14	1.842
20	1.921
30	2.017
40	2.089

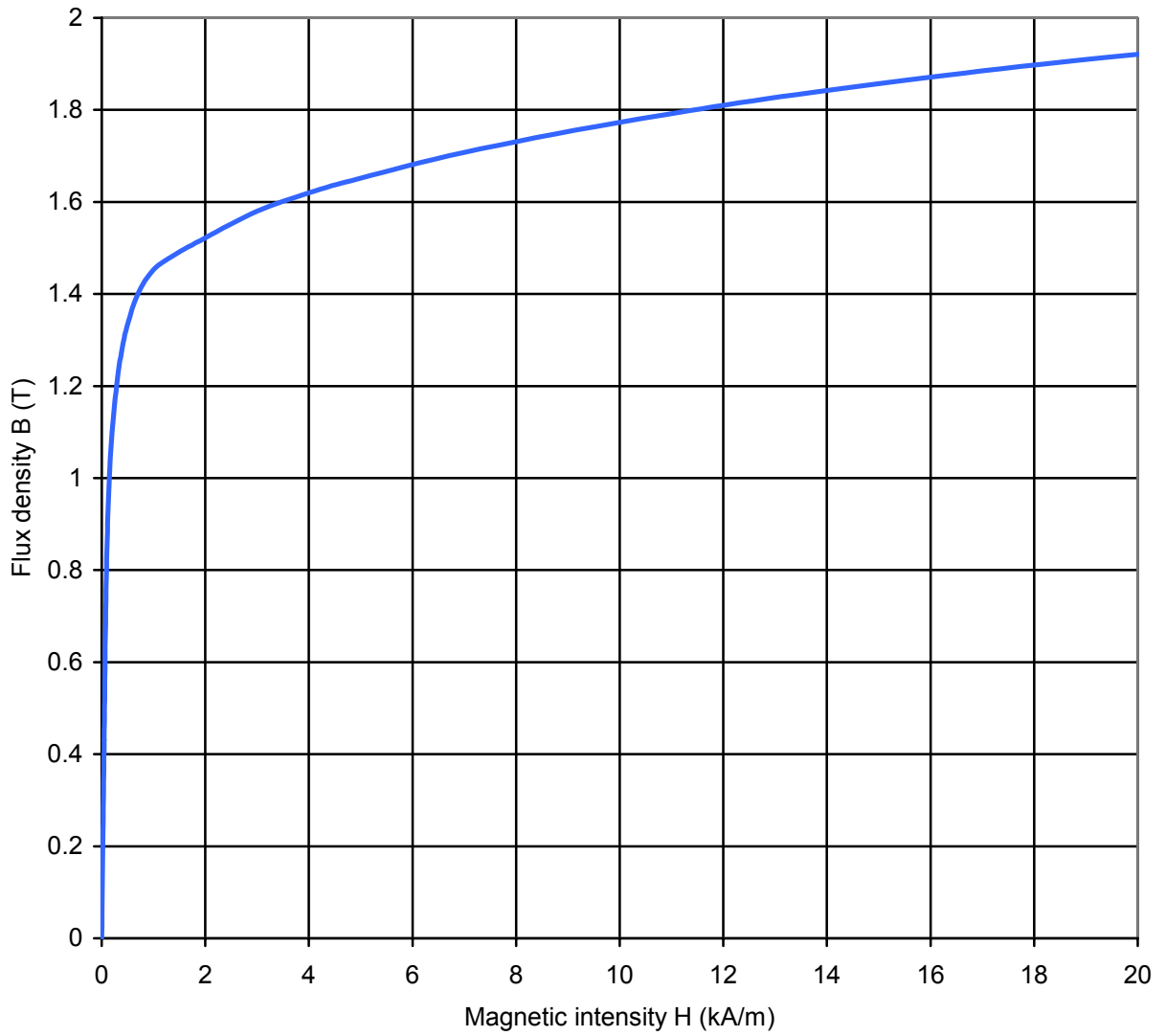


Figure 12-1: Carpenter silicon steel: extended characteristic

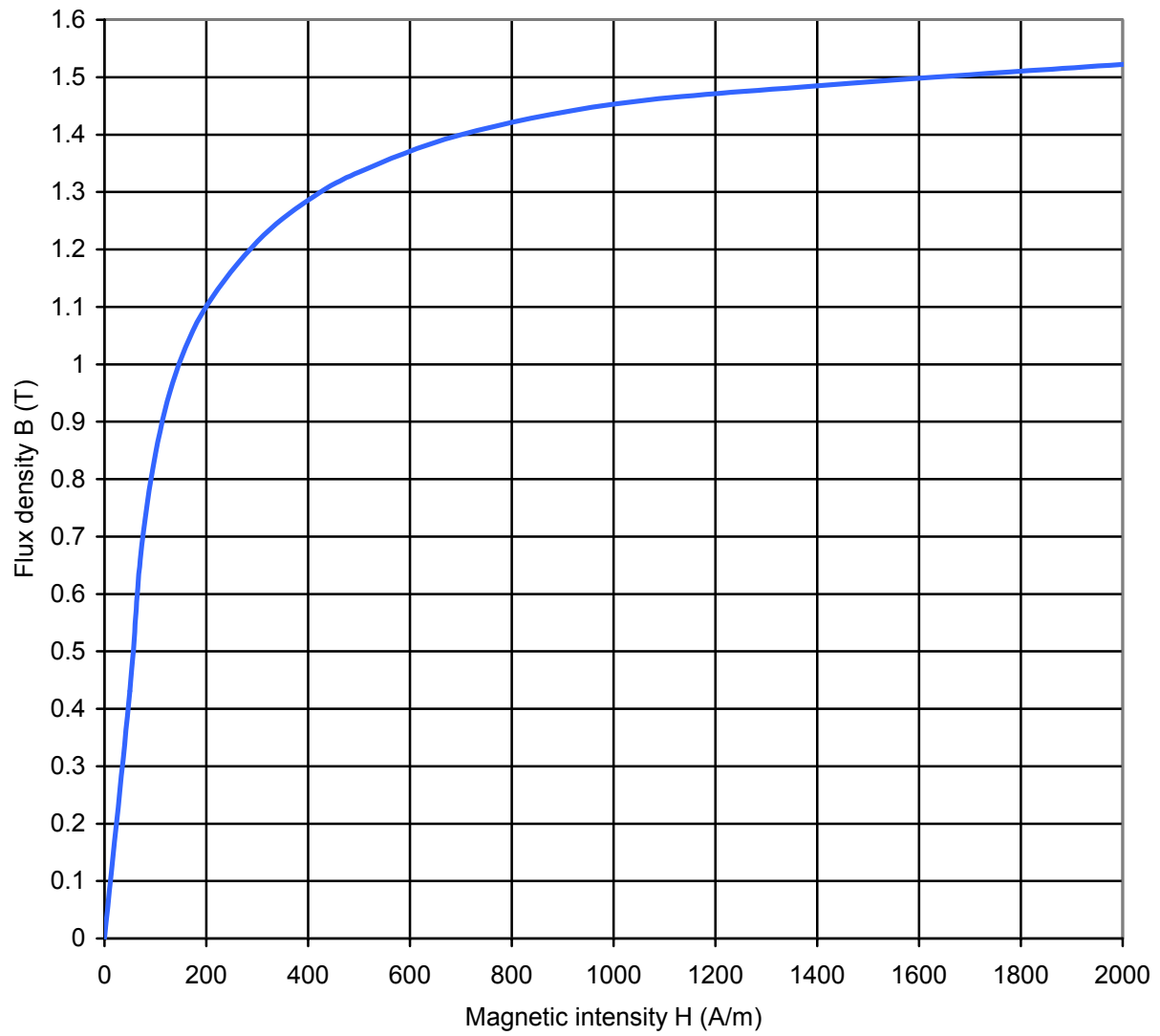


Figure 12-2: Carpenter silicon steel: expanded characteristic

12.2 List of formulae

Magnetic force on a current element: $\mathbf{df} = i\mathbf{dl} \times \mathbf{B}$

Ampère's circuital law: $\oint \mathbf{H} \cdot d\mathbf{l} = \sum i = Ni$

Materials:

$$B = B(H) \approx \mu_r \mu_0 H, \quad \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Magnetic flux and flux linkage: $\phi = BA, \quad \lambda = N\phi$

Faraday's law: $e = \frac{d\lambda}{dt} = N \frac{d\phi}{dt} = \frac{d}{dt}(Li)$

Inductance: $L = \frac{\lambda}{i}, M = \frac{\lambda_{21}}{i_1} = \frac{\lambda_{12}}{i_2}$

Coupled coils:

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Stored energy: $W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$

MMF: $\mathbf{F} = Ni = \sum Hl = \mathbf{R}\phi$

Reluctance: $\mathbf{R} = \frac{Ni}{\phi} = \frac{l}{\mu_r \mu_0 A}$

Resistance: $R = \frac{\rho l}{A} = \frac{l}{\sigma A}$

Force: $f_x = \frac{1}{2} i^2 \frac{\partial L}{\partial x} = -\frac{1}{2} \phi^2 \frac{\partial \mathbf{R}}{\partial x}$

Torque: $T_\theta = \frac{1}{2} i^2 \frac{\partial L}{\partial \theta} = -\frac{1}{2} \phi^2 \frac{\partial \mathbf{R}}{\partial \theta}$

Magnetic force per unit area: $\frac{f_m}{A} = \frac{1}{2} \frac{B^2}{\mu_0}$

Transformers – voltage:

$$v_1 \approx N_1 \frac{d\phi}{dt}, \quad v_2 \approx N_2 \frac{d\phi}{dt}$$

Transformers – AC operation:

$$V_{1m} = N_1 \omega \Phi_m = 2\pi f N_1 \Phi_m = 2\pi f N_1 A B_m$$

Transformers – current: $N_1 i_1 - N_2 i_2 = \mathbf{R}\phi \approx 0$

Ideal transformer: $\frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{i_2}{i_1}$

Moving-coil transducers:

$$e = Blu, \quad f = Bli$$

$$L = (Bl)^2 K, \quad R = \frac{(Bl)^2}{D}, \quad C = \frac{M}{(Bl)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{MK}}$$

DC machines:

$$e_a = K\phi_f \omega_r, \quad T_d = K\phi_f i_a$$

$$e_a \approx K' i_f \omega_r, \quad T_d \approx K' i_f i_a$$

Armature equation: $v_a = R_a i_a + e_a = R_a i_a + K\phi_f \omega_r$

Speed control: $v_a \approx e_a = K\phi_f \omega_r$

Series motor: $i_f = i_a = i$

Star connection: $|V_{line}| = \sqrt{3} |V_{phase}|$

Delta connection: $|I_{line}| = \sqrt{3} |I_{phase}|$

3-phase power: $P = \sqrt{3} V_{line} I_{line} \cos \phi$

Rotating field ($2p$ poles): $\omega_s = \frac{2\pi f}{p}$ [rad/s],

$n_s = \frac{f}{p}$ [rev/s], $N_s = 60n_s = \frac{60f}{p}$ [rev/min].

Synchronous motor: $N_r = N_s$

Induction motor: $N_r < N_s$

Slip speed: $N_s - N_r$

Fractional slip: $s = \frac{N_s - N_r}{N_s} = \frac{\omega_s - \omega_r}{\omega_s}$

Rotor input: $P_{em} = \omega_s T_d$

Rotor output: $P_{out} = \omega_r T_d = (1-s)\omega_s T_d$

Rotor loss: $P_{loss} = (\omega_s - \omega_r) T_d = s\omega_s T_d$