Convection displacement current and alternative form of Maxwell-Lorentz equations

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Abstract

Some mathematical inconsistencies in the conventional form of Maxwell's equations extended by Lorentz for a single charge system are discussed. To surmount these in framework of Maxwellian theory, a novel convection displacement current is considered as additional and complementary to the famous Maxwell displacement current. It is shown that this form of the Maxwell-Lorentz equations is similar to that proposed by Hertz for electrodynamics of bodies in motion. Original Maxwell's equations can be considered as a valid approximation for a continuous and closed (or going to infinity) conduction current. It is also proved that our novel form of the Maxwell-Lorentz equations is relativistically invariant. In particular, a relativistically invariant gauge for quasistatic fields has been found to replace the non-invariant Coulomb gauge. The new gauge condition contains the famous relationship between electric and magnetic potentials for one uniformly moving charge that is usually attributed to the Lorentz transformations. Thus, for the first time, using the convection displacement current, a physical interpretation is given to the relationship between the components of the four-vector of quasistatic potentials. A rigorous application of the new gauge transformation with the Lorentz gauge transforms the basic field equations into an independent pair of differential equations responsible for longitudinal and transverse fields, respectively. The longitudinal components can be interpreted exclusively from the standpoint of the instantaneous "action at a distance" concept and leads to necessary conceptual revision of the conventional Faraday-Maxwell field. The concept of electrodynamic dualism is proposed for self-consistent classical electrodynamics. It implies simultaneous coexistence of instantaneous long-range (longitudinal) and Faraday-Maxwell short-range (transverse) interactions that resembles in this aspect the basic idea of Helmholtz's electrodynamics.

I. INTRODUCTION

In the early part of 19'th century, the highest importance was attached to electricity and magnetism in an attempt to justify a particular outlook on the world. After Faraday's fundamental discovery of the law of induction, the challenge presented itself of unifying electrodynamics into a coherent whole out of the electrostatics of Neumann and magnetostatics of Ampère. From the historical point of view it was an exiting and unique moment in the development of physics [1]. Two rival concepts: instantaneous action at a distance and Faraday's field concept were waiting for decisive progress in theory and experiment: to be confirmed or rejected. The state of electromagnetism was characterized by the search for a correct and unambiguous concept by excluding all alternatives. In 1848, Wilhelm Weber was the first to attempt to unify electromagnetic theory [2]. At the time, Weber's action at a distance type theory was felt to be a great advance because it reproduced the results of both Neumann and Ampère using an analytical treatment of induced currents. Some years later in 1855-56, James Clerk Maxwell communicated to the Cambridge Philosophical Society his first memoir: an attempt to develop a comprehensive mechanical conception of the electromagnetic field. This afterwards developed into a powerful field theory [3,4]. In the years immediately following the Maxwell's *Treatise*, a certain amount of evidence in favor of his theory was furnished by experiment. Nevertheless, it was not sufficient to make a choice between the two alternatives. Two theories; those of Weber and Maxwell, could satisfactory describe the major known electromagnetic phenomena in spite of many internal limitations.

In 1870, in consequence of long and fruitless opposition of two rival concepts in electromagnetism, and for the purpose of reconciling them, a compromise theory was proposed by Helmholtz [5]. Soon it became the accepted theory in Germany and continental Europe. In particular, Hertz and Lorentz got to know the Maxwellian theory through Helmholtz's equations. The compromise theory of Helmholtz also was incomplete and internally inconsistent. It applied exclusively to a dielectric medium at rest and did not take into consideration displacement currents in vacuo (ether). Helmholtz's theory allowed simultaneous coexistence of instantaneous longitudinal electric modes and transverse electric and magnetic waves propagating with a certain finite spread velocity. The magnetic field, however, could be transmitted only through transverse modes. There was conceptual conflict with Maxwell's theory. For instance, the value of transverse wave velocity differed considerably from that predicted by the Maxwell field equations in a dielectric medium.

At that point in time there was an urgent need for reliable new experimental data. The Berlin Academy proposed as a prize subject "To establish experimentally a relation between electromagnetic action and the polarization of dielectrics". This investigation led Hertz to a discovery of great importance that is now widely accepted as a crucial moment in the history of electromagnetism. Hertz succeeded in directly observing the propagation of electromagnetic waves in free space with the velocity predicted by Maxwell and thereby decisively tested his theory. Another two alternatives (Weber's and Helmholtz's theories) had been rejected in spite of the fact that Helmholtz's theory, corrected for the spread velocity for transverse modes no longer contradicted the Hertz experiment. In the literature devoted to the history and methodology of physics there was great discussion whether one experiment such as of Hertz could be considered as a decisive argument at the choice between a few alternatives [6]. From here on, all main investigations in electromagnetism were based on Maxwell's equations. Nevertheless, this theory still suffered from some short comings inherent to its predecessors. To be more specific, Maxwell's equations for steady state processes could be compatible only with continuous, closed currents derived from Ampère's magnetostatics. The theory needed to be extended to a one charge system.

In 1881, the first examination of the matter from the standpoint of Maxwell's theory was undertaken by J.J. Thompson [7] and later by O. Heaviside [8]. They found that the change in the location of the charge (they considered an electrostatically charged body in motion) must produce a continuous alteration of the electric field at any point in the surrounding medium (the point of difference with Maxwell's theory of stationary fields). In the language adopted by Maxwell, there must be displacement currents in the vacuum attributed to the magnetic effects of moving charge. Developing this approach, FitzGerald pointed out in a short but valuable note [9] that Thompson's method must be identified with the basic hypothesis of Maxwell's theory about the total current. This conclusion was based on the assumption that a moving charge itself was to be counted as a current element. Then the total current, thus composed of the displacement currents and a moving charge, was circuital, in accordance with Maxwell's fundamental ideas.

As the result of their investigations, Thompson and Heaviside found the electric and magnetic fields of a moving charge that now can be verified directly by use of Lorentz transformations. They also gave for the first time an explicit formula for mechanical force acting on an electric charge which is moving in a magnetic field, now known as the Lorentz force. It was the first successful demonstration that Maxwell's theory might be extended to a one charge system.

In spite of the advances which were effected by Maxwell and his earliest followers a more general theory was needed for bodies in motion. In 1890, Hertz made the first attempt to build up such a theory. As in all 19'th century methods, Hertz based his considerations on an adequate model of ether. As Hertz himself expressed it, "the ether contained within ponderable bodies moved with them". Like Maxwell, he assumed that the state of the compound system (matter and ether) could be specified in the same way when matter was in motion or at rest. Thus, it was Hertz who established for the first time the relativity principle in electrodynamics. It could not have been as general as the Einstein-Poincarè relativity principle since it was tied to a model of the ether. However, as will be shown in this work, Hertz's equations, carefully reconsidered, can be treated as a relativistically invariant and alternative form of Maxwell's equations. The results of Hertz's theory resembled in many respects those of Heaviside who likewise was disposed to accept an additional term in Maxwell's equations involving $curl(\mathbf{V} \times \mathbf{E})$ that they called the *current of dielectric convection*.

However, contemporaries of Hertz did not accept as altogether successful his attempts to extend the theory of the electromagnetic field to the case in which ponderable bodies are in motion. Hertz's assumptions on the ether were conceptually inconsistent with the existing interpretation of Fizeau's experiment and seemed to disagree with Fresnel's formulae. Meanwhile, in the decade of 1890, the electrodynamics of moving media was systematically treated by Lorentz. The principal differences by which the theory advanced by Lorentz was distinguished from Hertz's one lay in the conception of "electron" and in the model of the ether. Lorentz designed his equations in accord with the successful Fresnel theory. A distinction had been made between matter and ether by assuming that a moving ponderable body could not communicate its motion to the ether which surrounds it. This hypothesis corresponded to the ether in rest and implied that no part of the ether could be in motion relative to any other part. The correctness of Lorentz's hypothesis, as opposed to that of Hertz's one, was afterwards confirmed by various experiments, one of them was the discovery of the electron. The experimental confirmation that electric charges resided in atoms put the Lorentz theory at the center of scientific interest and made it the basis for all further investigations in the area of electromagnetism. The reconciliation of the electromagnetic theory with Fresnel's law of the propagation of light in moving bodies achieved in the framework of Lorentz's theory was considered as a distinct advance. Nevertheless, the theory was far from complete and posed various problems. Some of them were internal, others were related to the direct application of the theory.

The decisive consolidation of the Lorentz theory of bodies in motion was made by A. Einstein who established a fundamental relativity principle. Since then a more comprehensive and consistent scheme of electromagnetic phenomena has been available. This was a turning-point in the development of electromagnetism. The mechanistic conception of ether had been abolished. Fresnel's formulae could be treated now on the basis of the relativistic law of addition of velocities. The principal arguments against the Hertz's theory were removed. However, Lorentz's theory, conceptually renovated, (without any attempt to do the same with Hertz's theory) was retained as the basis of conventional classic electrodynamics. In spite of all advances, the electromagnetic theory of Lorentz has internal inconsistencies such as the self-reaction force (*self-interaction*), infinite contribution of self-energy, the concept of electromagnetic mass, indefiniteness in the flux of electromagnetic energy, unidirectionality of radiation phenomena with respect to motion reversal in the basic Maxwell equations etc. The advent of quantum mechanics in the early part of this century brought the hope that all classical difficulties could be straightened out in the framework of quantum electrodynamics. Quantum field theory does alleviate some problems but cannot surmount them without introducing unjustified renormalization methods. The principal difficulties in Maxwell's theory persist and do not disappear in spite of taking into account quantum mechanical modifications. The Hertz theory, renovated in accordance with Einstein's relativity principle, might provide a more satisfactory alternative.

As will be shown here, a modification of the Maxwell-Hertz equations can be made equivalent to a form of the Maxwell-Lorentz set of equations, and their rigorous solution shows the existence of a longitudinal, explicitly time independent, component of the electromagnetic field. This finding supports some recent investigations in the new area of longitudinal field solutions of the fundamental electromagnetic field equations proposed by M. W. Evans and J.-P. Vigier [10,11].

The general solution of our modified field equations reproduces the "separation of potentials" theory proposed recently in [12] as a method of removing the above-mentioned difficulties from classical electromagnetic theory. In [12] the conventional Faraday-Maxwell concept of the field was shown to be not fully adequate for Maxwell's equations.

Electrodynamical dualism was developed, and implies the coexistence of the instantaneous long-range and Faraday-Maxwell short-range interactions. For the first time since the Hertz's experimental discovery, an argument has been developed in favor of Helmholtz's alternative theory renovated in the framework of Maxwell's equations. The concept of action at a distance in this theory differs completely the action at a distance theories of postspecial relativity [13,14], theories which assume that only delayed action at a distance with the speed of light can be consistent with relativity. The new approach developed in [12] demonstrates the compatibility of instantaneous action at a distance with relativistic classical electrodynamics. The new "dualism concept" can bridge the gap between classical and quantum physics. From this point of view, the instantaneous action at a distance might become a classical analogy of non-locality in quantum theories.

Let us conclude here this historical background and go on to the analysis of some difficulties in the Lorentz electrodynamics.

II. MAXWELL-LORENTZ EQUATIONS. PARADOX

Let us write Maxwell's equations for the reference system at rest, grouping them into two pairs [15]:

$$div\, \mathbf{E} = 4\pi\varrho, \tag{1}$$

$$div\, \mathbf{B} = 0 \tag{2}$$

and

$$curl\, \mathbf{H} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{3}$$

$$curl\, \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{4}$$

at the same time with the continuity equation:

$$\frac{\partial \varrho}{\partial t} + div\, \mathbf{j} = 0. \tag{5}$$

In the phenomenological theory of electromagnetism the hypothesis about the continuous nature of medium was one of the foundations of Maxwell's speculations. This point of view succeeded in uniting all electromagnetic phenomena without the necessity to consider a specific structure of matter. Nevertheless, the macroscopic character of the charge conception defines all well-known limitations on Maxwell's theory. For instance, the system of electromagnetic fields equations (1-4) in a steady state corresponds to a quite particular case of continuous and closed conduction currents (motionless as a whole).

In 1895, the theory was extended by Lorentz for a system of charges moving in vacuo. Since then it has been widely assumed that the same basic laws are valid microscopically and macroscopically in the form of original Maxwell's equations. It means that in Lorentz form all macroscopic values of charge and current densities must be substituted by its microscopic values. Let us write explicitly the Lorentz field equations for one point charged particle moving in vacuo [15]:

$$div\,\mathbf{E} = 4\pi q\delta(\mathbf{r} - \mathbf{r}_q(t)), \tag{6}$$

$$div\,\mathbf{B} = 0 \tag{7}$$

and

$$curl\,\mathbf{H} = \frac{4\pi}{c} q\mathbf{V}\delta(\mathbf{r} - \mathbf{r}_q(t)) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \tag{8}$$

$$curl\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{9}$$

here $\mathbf{r}_q(t)$ is the coordinate of the charge at an instant t.

In order to achieve a complete description of a system consisting of fields and charges in the framework of electromagnetic theory, Lorentz supplemented (6)-(9) by the equation of motion of a particle:

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}(\mathbf{V} \times \mathbf{B}) \tag{10}$$

where $\mathbf{p}$ is the momentum of the particle.

The equation of motion (10) introduces an expression for the mechanical force known as Lorentz force which in electron theory formulated by Lorentz clearly has an axiomatic and empirical character. Later on we shall discuss some disadvantages related with the adopted status of the Lorentz force conception.

The macroscopic Maxwell's equations (1)-(4) may be obtained now from Lorentz's equations (6-9) by some statistical averaging process, using the structure of material media. The mathematical language for eqs. (6)-(9) is widely adopted in the conventional classical electrodynamics.

However, there is an ambiguity in the application of this equations to the case of one uniformly moving charge. Really, a simple charge translation in space produces alterations of field components. Nevertheless, they can not be treated in terms of Maxwell's displacement current. Strictly speaking, in this case all Maxwell's displacement currents proportional to $\partial\mathbf{E}/\partial t$ and $\partial\mathbf{B}/\partial t$ vanish from eqs. (8)-(9). This statement can be reasoned by two different ways: (*i*) $\partial\mathbf{E}/\partial t = 0$ and $\partial\mathbf{B}/\partial t = 0$, since all field components of one uniformly moving charge are implicit time-dependent functions (time enters as a unique parameter) so that from the mathematical standpoint only total time derivative can be applied in this case whereas partial time derivative turns out to be not adequate (time and distance are not independent variables); (*ii*) a non-zero value of $\partial\mathbf{E}/\partial t$ and $\partial\mathbf{B}/\partial t$ would imply a local variation of fields in time independently of the charge position and hence would imply the expansion of those local variations through the propagation of electromagnetic waves. This would contradict the fact that one uniformly moving charge does not radiate electromagnetic field.

In this respect, it will be shown in this work that in a mathematically consistent form of Maxwell-Lorentz set of equations all partial time derivatives must be substituted by *total* ones. Only in this way all ambiguities related to the application of Maxwell's displacement current can be removed. On the other hand, it would imply a correct extension of this concept to all quasistatic phenomena.

Thus, a mathematically rigorous interpretation of eqs. (8)-(9) in the case of a charge moving with a constant velocity leads to the following conclusion: in a free space out of a charge the value of $curl\,\mathbf{H}$ in eq. (8) is equal to zero:

$$curl\,\mathbf{H} = \frac{4\pi}{c} q\mathbf{V}\delta(\mathbf{r} - \mathbf{r}_q(t)) \tag{11}$$

On the other hand, the case of one uniformly moving charge has been studied in detail and can be treated exactly in the framework of Lorentz's transformations. Therefore, for any purpose one can apply exact mathematical expressions for electric and magnetic fields and potentials of a moving charge as follows [15]:

$$\mathbf{E} = q\frac{(\mathbf{R} - R\boldsymbol{\beta})(1-\beta^2)}{(R - \boldsymbol{\beta}\mathbf{R})^3}, \tag{12}$$

$$\mathbf{H} = \boldsymbol{\beta} \times \mathbf{E} \tag{13}$$

where $\boldsymbol{\beta} = \mathbf{V}/c$.

Thus, we arrive at an important conclusion: generally speaking, the value of $curl\,\mathbf{H}$ is *not* equal to zero in any point out of a moving charge and takes a well-definite quantity:

$$curl\,\mathbf{H} = \frac{1}{c}(\mathbf{V} \times \mathbf{E}). \tag{14}$$

For instance, it gives immediately a non-zero value of $curl\,\mathbf{H}$ along the direction of motion (X-axis):

$$curl_x\,\mathbf{H}(x > x_q) = q\frac{2\beta(1-\beta^2)}{(1-\beta)^3(x-x_q)^3}. \tag{15}$$

The conflict with the previous statement of equation (11) is inevitable. In order to obtain symmetry between the set of field equations (6)-(9) and their solutions one would expect an additional term like that considered in (14). As it will be shown, this assumption resembles in many respects the ingenious idea of Maxwell about the displacement current that had revealed a profound symmetry and interrelation between the electric and magnetic fields.

From the phenomenological point of view, the additional term could be understood in the following manner. Really, every charge during its motion changes the electric field flux through some fixed surface S bounded by the contour C. According to the Gauss' theorem, the same rate of electric flux through S might be produced from any other point of space. It would correspond to some effective charge moving at the instant with a certain velocity. In terms of Stokes' theorem this fact takes a clear form. The circulation of the magnetic field is related with the amount of the electric current passing through the surface of integration. Since all surfaces enclosed by the contour C are equivalent in accordance with the conditions of Stokes' theorem then one could reasonably assume the existence of *effective current* in the space out of a charge that would make the same contribution into integral:

$$\oint\limits_C \mathbf{H}\,d\mathbf{l} = \int\limits_{S'} curl\,\mathbf{H}\,d\mathbf{S} = \frac{4\pi}{c}\int\limits_S \mathbf{j}d\mathbf{S} \tag{16}$$

where S' and S are equivalent surfaces bounded by C.

In the case of uniformly moving charge, this *effective current* play the same role as the displacement current introduced by Maxwell for non-steady processes. As it is well-known, the introduction of Maxwell's displacement current was made on the base of following formal reasoning. In order to make the equation (3) consistent with the electric charge conservation law in form of continuity equation (5), Maxwell supplemented (3) with an additional term. However, for stationary processes this term disappears and the equation (3) becomes consistent only with closed (or going off to infinity) currents:

$$div\,\mathbf{j} = 0. \tag{17}$$

On the other hand, this is a direct consequence of the continuity equation (5) in any stationary state when all magnitudes must be considered as implicit time-dependent functions. Thereby, we meet here another difficulty of Lorentz's equations: uniform movement of a single charged particle (as an example of *open steady current*), generally speaking, does not satisfy the limitation imposed by equation (17). It implies some additional term to be taken into consideration in (17) to fulfill the Maxwell's condition on the circuital character of the total current. A detailed analysis of the matter will be given in the next section.

Let us make a final remark about the following denomination that will be adopted throughout this work. Further, we shall distinguish so-called quasistatic fields among a more general class of common time-varying fields. The study of steady processes in the original Maxwell's theory is reduced to static fields when there is no change with time. It is possible to overcome that limitation if one contrasts field alterations with time in the case of uniformly moving charge from usual time-dependent fields. As a matter of fact, it occupies an intermediate place between purely static and common non-steady processes since all field values keep an implicit time-dependence if the motion is uniform. This is reason to treat such a case as quasistatic (or quasistationary) by definition.

III. BALANCE EQUATION AND ALTERNATIVE FORM OF MAXWELL-LORENTZ EQUATIONS FOR A SINGLE CHARGED PARTICLE

As it was pointed out in the *Introduction*, one of the principle problems in the original Maxwell's theory was the relation between the charge and matter. In fact, for many years Maxwell was adhered to the Faraday's idea that the charge is a sort of field manifestation and at first time directed all his efforts in creating an unified theory of matter as a field theory. However, further Maxwell adopted another position in this respect by clear distinguishing between fields and matter as two supplementary conceptions. Since then a charge had been entirely attributed to the field and therefore opposed to matter. Finally, this ambiguous approach constituted the foundation of so-called "*operative interpretation*" of Maxwell's field theory without any profound assumptions on the charge nature and its relation with matter. In any case it may be explained by the questionable status of many experimental data about the charge at the middle of 19'th century. Fascinated by Faraday's idea of tubes of field, Maxwell adopted this analogy also for electric and magnetic fields as well as for electric charge flow (conduction currents). As a consequence, in accordance with hydrodynamics language, the continuity equation was accepted as valid to express the hypothesis that a net sum of electric charges could not be annihilated. In this case, the continuity equation reproduces the charge conservation law in the given fixed volume $\mathcal{V}$:

$$\frac{d\mathcal{Q}}{dt} = \int\limits_{\mathcal{V}} \left(\frac{\partial \varrho}{\partial t} + div\,\mathbf{j} \right) d\mathcal{V} = 0 \tag{18}$$

or in form of differential equation:

$$\frac{\partial \varrho}{\partial t} + div\,\mathbf{j} = 0 \qquad\qquad \left(\frac{d\mathcal{Q}}{dt} = 0 \right). \tag{19}$$

It should be remarked that eq. (19) describes exclusively the conservation but not the change of the amount of charge (or matter) in the given volume $\mathcal{V}$ as one can meet in some or monographs that sometimes make no distinction between this two aspects. If one wants to describe the change of something in given volume $\mathcal{V}$, must replace the equation (18) by a balance equation (see, e.g. [16]):

$$\frac{d\mathcal{Q}}{dt} = \frac{d}{dt} \int\limits_{\mathcal{V}} \varrho d\mathcal{V} = - \oint\limits_{S} \mathbf{j} d\mathbf{S} = - \int\limits_{\mathcal{V}} div\,\mathbf{j}\, d\mathcal{V}. \tag{20}$$

Here $\mathbf{j}$ is a total current of electric charges through a surface S that bounds the given volume $\mathcal{V}$. In mathematical language common to all physical theories it means that the rate of increase in the total quantity of electrostatic charge within any fixed volume is equal to the excess of the influx over the efflux of current through a closed surface S. On contracting the surface to an infinitesimal sphere around a point one can arrive at the differential equation [16]:

$$\frac{d\varrho}{dt} + div\,\mathbf{j} = 0 \qquad\qquad \left(\frac{d\mathcal{Q}}{dt} \neq 0\right). \tag{21}$$

The balance equation (21) includes the continuity equation (19) as a particular case when amount of something (charge or matter) maintains constant in $\mathcal{V}$ with the course of time. In the previous section we remarked that a single charge in motion, generally speaking, could not be treated in terms of continuity equation (19) unless one is using so-called *slow motion approximation* [17]. By that condition is understood to be a motion of charges in a certain limited region which charges do not leave during the time of observation. Hence the equation (17) for one uniformly moving charge is valid only under this condition. In more general case, when a particle may leave the volume and thereby, violate locally the charge conservation, one should be addressed to the balance equation (21). One simple method is to prove it on an example of point-charge moving with a constant velocity. In particular, the charge density is assumed to have implicit time dependence as follows:

$$\varrho(\mathbf{r}, \mathbf{r}_q(t)) = q\delta(\mathbf{r} - \mathbf{r}_q(t)) \tag{22}$$

where $\mathbf{r}_q(t)$ is the coordinate of the charge at an instant t.

It is easy to show that the total density derivative in respect to time will contain only the convection term, since the time enters in eq. (22) as parameter ($\partial\varrho/\partial t = 0$):

$$\frac{d\varrho}{dt} = \frac{\partial\varrho}{\partial t} + \left(\left\{\frac{d}{dt}(\mathbf{r} - \mathbf{r}_q(t))\right\} \cdot \, grad\,\varrho\right) = -(\mathbf{V} \cdot \, grad\,\varrho) \tag{23}$$

where $\mathbf{V} = d\mathbf{r}_q/dt$ is the velocity of the charge at the instant t.

Thus, the balance equation for a single particle is fulfilled directly:

$$-(\mathbf{V} \cdot \, grad\,\varrho) + div(\varrho\mathbf{V}) = -(\mathbf{V} \cdot \, grad\,\varrho) + (\mathbf{V} \cdot \, grad\,\varrho) = 0. \tag{24}$$

In spite of disappointing ambiguity of the original charge conception, all direct followers of Maxwell (Thompson, Heaviside, FitzGerald and Pointing) did not tried to make any additional assumptions on the relationship between matter and charge. At first place, they were interested to establish a true relation between a charge in uniform motion and a field that surrounds it. The essence of their method was the Thompson's theory of moving tubes of electric and magnetic forces. Without any explicit use of continuity equation Thompson deduced that there must be displacement currents in the space outside the sphere of electrostatically charged body [7]. The further advance was effected by FitzGerald who had associated that conclusion with the Maxwell's position in respect to circuital character of the total electric current. In other words, the total current of uniformly moving electrostatically charged body was to be composed of the displacement current in outer space and the motion of body itself so that the following condition might hold:

$$div(\mathbf{j}_{\mathtt{cond}} + \mathbf{j}_{\mathtt{disp}}) = 0 \tag{25}$$

where $\mathbf{j}_{\mathtt{cond}}$ and $\mathbf{j}_{\mathtt{disp}}$ are conduction and displacement currents respectively.

Here we shall not enter in details about the results obtained by Thompson and et. al. but only mention that their approach for the first time demonstrated the fundamental role of the displacement current (which was entirely due to the translation of a charge and not to the explicit time-varying processes) in the interrelation between quasistatic electric and magnetic fields. The situation was so peculiar that Heaviside himself insisted on the necessity to supplement Maxwell's equations with an additional term. He assured that this term was so in the spirit of the Maxwell's theory that if somebody had proposed this modification to Maxwell, he would accepted it without any hesitation. From this position, it seems that only balance equation in the case of uniformly moving charge may be in accordance with the general Maxwell's condition (25) whereas the continuity equation leads to the limited condition imposed by (17).

Really, we can rewrite the eq. (21) in the form of eq. (25) taking into account the following denomination:

$$div\,\mathbf{j}_{\mathtt{disp}} = \frac{d\varrho}{dt} = \frac{d}{dt}\left(\frac{1}{4\pi}\,div\,\mathbf{E}\right) = div\left(\frac{1}{4\pi}\frac{d\mathbf{E}}{dt}\right) \tag{26}$$

It may be easily verified that two operations div and $\frac{d}{dt}$ are completely interchangeable (for $div\,\mathbf{V} = 0$) in (26). Thus, in a particular case of an arbitrary moving charge when one can disregard its size, the Maxwell's condition on total displacement current takes the following form (see for the sake of comparison the formula (23)):

$$div\,\mathbf{j}_{\mathtt{disp}} = \frac{1}{4\pi}div\left\{\frac{\partial\mathbf{E}}{\partial t} - (\mathbf{V}\cdot\nabla)\mathbf{E}\right\} \tag{27}$$

So far we make use of the formal mathematical approach without any physical interpretation. More specific, in calculating the full derivative of $\mathbf{E}$ in respect to time, the convection term in (27) must be considered as quasistationary (fixed time condition) in agreement with the definition of partial derivatives. In correct mathematical language it means as if all field alterations produced by a simple charge translation occur instantaneously (at same instant of time) *in every space point.* This interpretation has no precedents in the classical electrodynamics for the case of arbitrary moving charge whereas for an uniform movement this quasistationary description is the *only possible* formalism.

In this connection, recently, a validity of so-called *quasistationary approximation* for, at least, one part of field quantities every instant of time has been demonstrated in [12]. It is understood to be a direct consequence of the mathematical condition on the continuity of general solutions of the set of Maxwell's equations. In particular, for uniformly moving charge, this approximation can be extended to all field values. Indeed, in this case, all field alterations due to charge translation in space occur instantaneously in every point, in accordance with the conventional point of view. Thus it is assumed that all field quantities can be separated into *two independent classes* with explicit $\{\}^*$ and implicit $\{\}_0$ time dependence, respectively. The component $\mathbf{E}_0$ of the total electric field $\mathbf{E}$ in every point is understood to depend *only* on the position of source at a given instant. In other words, $\mathbf{E}_0$ is rigidly linked with the location of the charge. From this point of view, the partial time derivative in (27) must be related *only* with the explicit time dependent component $\mathbf{E}^*$ whereas the convection derivative *only* with $\mathbf{E}_0$.

$$\frac{d\mathbf{E}}{dt} = \frac{\partial \mathbf{E}^*}{\partial t} - (\mathbf{V}\cdot\nabla)\mathbf{E}_0; \qquad \text{where} \qquad \mathbf{E} = \mathbf{E}^*(\mathbf{r},t) + \mathbf{E}_0(\mathbf{R}(t)) \tag{28}$$

($\mathbf{r}$ is a fixed distance from the origin of the reference system at rest to the point of observation; $\mathbf{R}(t) = \mathbf{r} - \mathbf{r}_q(t)$; $\mathbf{r}_q(t)$ and $\mathbf{V} = \frac{d\mathbf{r}_q}{dt}$ are the distance and the velocity of an electric charge.

In spite of this apparent relationship with the results developed in [12], they have been advocated here exclusively for justifying the formal mathematical use of the total time derivative in form of (27). As a matter of fact, the separation of total field quantities into $\mathbf{E}_0$ and $\mathbf{E}^*$ is obtained here independently and turns out to be the exact mathematical result related to the existence of two non-reducible (independent) parts in the expression of total time derivative. However, later on, we shall use same denomination of field values in this two parts of (28), always taking into account respective additional conditions. Only in the last section we shall apply explicit separation of fields in basic equations in order to find respective gauge transformations.

Thus, a general expression of full displacement current is then given by the formulae:

$$\mathbf{j}_{\mathtt{displ}} = \frac{1}{4\pi}\frac{\partial \mathbf{E}}{\partial t} - \frac{1}{4\pi}(\mathbf{V}\cdot\nabla)\mathbf{E}. \tag{29}$$

Our initial aim was to find a reasonable form for the Maxwell's circuital condition (25) that would allowed to relate field alterations in free space produced by a single-moving charge with the Maxwell's conception of displacement current. From the standpoint of conventional classical electrodynamics, the first term represents the famous Maxwell's displacement current appearing only in not-steady processes whereas the second one could be understood as quasistationary term due to a simple charge translation in space. Further, it would be convenient to denominate this term as "*convection displacement current*"

The above results motivate an important extension of displacement current concept in the entire spirit of the Maxwellian original electromagnetic theory. First, it postulates the circuital character of the total electric current. Second, it permits to fulfill the circuital condition for not-steady as well as for steady processes (static and quasistatic fields), contrary to the conventional approach. Thus, we arrive independently to Heaviside's intention of supplementing Maxwell's equations with additional terms. As the last step before doing it, let us give an equivalent mathematical expression of the convection displacement current (in the case of single charged particle):

$$\frac{1}{c}(\mathbf{V}\cdot\nabla)\mathbf{E} = \frac{1}{c}\,\mathbf{V}\,div\,\mathbf{E} - \frac{1}{c}\,curl\,(\mathbf{V}\times\mathbf{E}). \tag{30}$$

Accordingly, for our purpose we need to remind that in the right part of eq. (8) the total current ($\mathbf{j}_{\mathtt{(tot)}} = \mathbf{j}_{\mathtt{(cond)}} + \mathbf{j}_{\mathtt{(disp)}}$ must be considered like this:

$$curl\,\mathbf{H} = \frac{4\pi}{c}q\mathbf{V}\delta(\mathbf{R}(t)) + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{1}{c}\mathbf{V}\,div\,\mathbf{E} + \frac{1}{c}curl(\mathbf{V}\times\mathbf{E}). \tag{31}$$

For completeness it is now necessary to assume that a moving charge may also possess a certain quantity of magnetic moment. Despite the difference between electric and magnetic charge conception we can usefully limit our consideration with the magnetic analogy of balance equation. This fortunate circumstance allows the treatment of eq. (9) in the same way as (8):

$$curl\,\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \frac{1}{c}(\mathbf{V}\cdot\nabla)\mathbf{B} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} + \frac{1}{c}\mathbf{V}\,div\,\mathbf{B} - \frac{1}{c}curl(\mathbf{V}\times\mathbf{B}). \tag{32}$$

Turning to the previous section where some inconsistencies of Lorentz's theory has been exposed, we see now that $curl\,\mathbf{H}$ is defined by (31) in every point out of a charge in the expected manner (see eq.(14)). As a final remark, eqs. (31,32) can be regarded as an alternative form of Maxwell-Lorentz system of field equations (6)-(9). In next section they will be analyzed on self-consistency and compared with modified Maxwell-Hertz equations extended on one charge system.

IV. ALTERNATIVE MAXWELL-HERTZ EQUATIONS FOR ONE CHARGE SYSTEM

Independently from Heaviside, the problem of modification of Maxwell's equations for bodies in motion was posed by Hertz in his attempts to build up a comprehensive and consistent electrodynamics [18]-[19]. A starting point of that approach was the fundamental character of Faraday's law of induction represented at first time by Maxwell in the form of integral equations:

$$\oint_C \mathbf{H}\,d\mathbf{l} = \frac{4\pi}{c}\int_S \mathbf{j}\,d\mathbf{S} + \frac{1}{c}\frac{d}{dt}\int_S \mathbf{E}\,d\mathbf{S} \tag{33}$$

$$\oint_C \mathbf{E}\,d\mathbf{l} = -\frac{1}{c}\frac{d}{dt}\int_S \mathbf{B}\,d\mathbf{S} \tag{34}$$

where C is a counter and S is a surface bounded by C.

In qualitative physical language Faraday's observations had been expressed in form of the following statement: *the effect of magnetic induction in the circuit C takes place everything with the change of the magnetic flux through the surface S independently weather it relates to the change of intensity of adjacent magnet or occurs due to the relative motion.* More over, Faraday established that the same effect was detected in a circuit at rest as well as in that at motion. The latter fact provided a principal base of Hertz's relativity principle which formulation resembles in some aspects that of Einstein. In order to avoid details of Hertz's original investigations [18]-[19], let us only note its sameness with traditional non-relativistic treatment of integral form of Faraday's law [20]. Namely, if the circuit C is moving with a velocity $\mathbf{v}$ in some direction, the total time derivative in

(33)-(34) must take into account this motion (convection derivative) as well as the flux changes with time at a point (partial time derivative) [20]:

$$\oint_C \mathbf{E}\, d\mathbf{l} = -\frac{1}{c}\frac{d}{dt}\int_S \mathbf{B}\, d\mathbf{S} = -\frac{1}{c}\left\{\frac{\partial}{\partial t} + (\mathbf{v}\cdot\nabla)\right\}\int_S \mathbf{B}\, d\mathbf{S} \tag{35}$$

where C and S are any circuit and surface bounded by C, respectively, moving together with a medium.

This approach is valid only for non-relativistic consideration and leads to Galilean field transformation [20]. It was just one of the problem which forced the complete rejection of Hertz's theory. More over, there was no alternative in the framework of the respective ether model used by Hertz. Any motion of the ether relative to the material particles had not been taken into account, so that moving bodies were regarded simply as homogeneous portions of the medium distinguished only by special values of electric and magnetic constants. Among the consequences of such assumption, Hertz saw the necessity to move the surface of integration in eqs. (33)-(34) at the same time with the moving medium. Thus, the generation of magnetic (or electric) force within a moving dielectric was calculated with implicit use of Galilean invariance in eq. (35) unless one makes any additional assumption on the special character of transformations in a moving frame of reference.

A persistent experimental search for direct proofs of the existence of the hypothetical ether led to the conclusion that its motion was completely undetectable. The discovery of the special relativity principle by Einstein that abolished at all the mechanistic conception of ether, gave rise to a serious objection to Hertz's approach. However, in the following we shall see a way to avoid those difficulties in the framework of the integral formulation of Maxwell's equation.

Let us now examine the case of a point source of electric and magnetic fields.[1] In order to abstain from the use of moving counter C and surface S that implies a direct application of the relativity principle, we limit our consideration to a fixed region (C and

[1]It should be noted that from mathematical standpoint of the developed approach there is no matter to have a detailed information about the nature of field source. Indeed, it may be a single charge, dipole, a constant magnetic moment etc that generates a quasistatic component $\mathbf{E}_0$ (or $\mathbf{B}_0$) of the total field $\mathbf{E}$ (or $\mathbf{B}$) in a given point and at an instant.

S are at rest) whereas the source is moving through a free space. Accordingly to Faraday's law, there must be an electromotive force in counter C due to the flux changes with time as well as with the motion itself. Generally speaking, this formulation must treat the time and convection derivatives simultaneously. In a strict mathematical language the convection part of the total derivative is to be considered at fixed time condition (or is to be calculated for every point at the same time) and, therefore, depends only on the position and the velocity of the point field source at an instant. We have previously mentioned the validity of the so-called *quasistationary approximation* for one part of field derivatives that in the case of uniformly moving charge may be extended to all field quantities. In other words, in the framework of the present integral approach a treatment of the total time derivative can be justified as analogous to differential form (28):

$$\frac{d\Phi}{dt} = \frac{\partial \Phi^*}{\partial t} - (\mathbf{V}_s \cdot \nabla)\Phi_0 \tag{36}$$

making use of the definitions:

$$\Phi_0^{(E)} = \int\limits_S \mathbf{E}_0(\mathbf{r} - \mathbf{r}_s(t))\, d\mathbf{S} \qquad \text{or} \qquad \Phi_0^{(B)} = \int\limits_S \mathbf{B}_0(\mathbf{r} - \mathbf{r}_s(t))\, d\mathbf{S} \tag{37}$$

and

$$\Phi^{*(E)} = \int\limits_S \mathbf{E}^*(\mathbf{r}, t)\, d\mathbf{S} \qquad \text{or} \qquad \Phi^{*(B)} = \int\limits_S \mathbf{B}^*(\mathbf{r}, t)\, d\mathbf{S} \tag{38}$$

where $\mathbf{r}$ is a fixed distance from the origin of the reference system at rest to the point of observation; $\mathbf{r}_s(t)$ and $\mathbf{V}_s = d\mathbf{r}_s/dt$ are distance and the velocity of the electric (or magnetic) field source.

For the sake of simplicity we shall conserve in this section the same denomination of field flux in two independent parts of total time derivative (36) taking into account tacitly additional (fixed space and fixed time) conditions, respectively, in the following expression:

$$\frac{d}{dt}\Phi = \left\{ \frac{\partial}{\partial t} - (\mathbf{V}_s \cdot \nabla) \right\} \Phi. \tag{39}$$

Using a well-known representation of the convection part in eq. (36) like that:

$$(\mathbf{V} \cdot \nabla) \int\limits_S \mathbf{E}\, d\mathbf{S} = \int\limits_S \mathbf{V}\, div\, \mathbf{E}\, d\mathbf{S} - \int\limits_S curl(\mathbf{V} \times \mathbf{E})\, d\mathbf{S} \tag{40}$$

we obtain an alternative form of Maxwell integral equations (33)-(34) for a moving electric charge in the reference system at rest:

$$\oint_C \mathbf{H}\, d\mathbf{l} = \frac{4\pi}{c}\int_S \mathbf{j}\, d\mathbf{S} + \frac{1}{c}\int_S \left\{\frac{\partial \mathbf{E}}{\partial t} - \mathbf{V}\, div\, \mathbf{E} + curl(\mathbf{V}\times\mathbf{E})\right\} d\mathbf{S} \tag{41}$$

$$\oint_C \mathbf{E}\, d\mathbf{l} = -\frac{1}{c}\int_S \left\{\frac{\partial \mathbf{B}}{\partial t} + curl(\mathbf{V}\times\mathbf{B})\right\} d\mathbf{S}. \tag{42}$$

Likewise (31)-(32), it is also assumed a single charge to have additionally a certain amount of a constant magnetic moment, i.e. to be a source of electric and magnetic fields at the same time.

Before passing over to the more general consideration of a large number of sources, we want to put attention to the most compact alternative differential form of Maxwell-Hertz equations in the reference system at rest:

$$div\, \mathbf{E} = 4\pi\varrho \tag{43}$$

$$div\, \mathbf{B} = 0 \tag{44}$$

$$curl\, \mathbf{H} = \frac{4\pi}{c}\varrho\mathbf{V} + \frac{1}{c}\frac{d\mathbf{E}}{dt} \tag{45}$$

$$curl\, \mathbf{E} = -\frac{1}{c}\frac{d\mathbf{B}}{dt} \tag{46}$$

where the total time derivative of any vector field value $\mathbf{E}$ (or $\mathbf{B}$) can be calculated by the following rule:

$$\frac{d\mathbf{E}}{dt} = \frac{\partial \mathbf{E}}{\partial t} - (\mathbf{V}\cdot\nabla)\mathbf{E}. \tag{47}$$

For the first time the above-mentioned form (43)-(46) was adopted by Hertz for electrodynamics of bodies in motion [18]-[19]. The only difference consisted in the definition of the total time derivative that corresponded to the moving medium (see (35)). A substitution of partial time derivative in (1)-(4) by total derivatives in respect to time in the alternative field equations (43)-(46) expresses the idea developed in [12] (and supported independently in the present approach) that a full solution of complete and consistent set of Maxwell's equations must be formed at the same time by *implicit and explicit time-dependent terms*. As it was yet mentioned, this fact is taken into account automatically in (43)-(46) by the simultaneous coexistence of two independent and mutually supplementary parts of total time derivative.

On the other hand, this alternative form of field equations removes the ambiguity related with the application of the Maxwell's displacement current concept in the case of a charge moving with a constant velocity. Any alterations of field components in space must be treated in this case exclusively in terms of the convection part of the total time derivative (47) whereas partial derivative in respect to time vanishes and hence is not adequate mathematically that confirms the analysis of some inconsistencies in the form of Maxwell-Lorentz equations given in the previous sections.

There is no difficulty to extend this approach to many particle system, assuming the validity of electrodynamics *superposition principle*. This extension is important in order to find out weather the alternative form of microscopic field equations contains the original (macroscopic) Maxwell's theory as limit case. To do that one ought to have into account all principal restrictions of Maxwell's equations (1)-(5) which have only dealing with continuous and closed (or going off to infinity) conduction currents. They also must be motionless as a whole (static tubes of charge flow), admitting only the variation of current intensity.

Under these conditions, it is quite easy to see that total (macroscopic) convection displacement current is canceled all by itself by summing up all microscopic contributions:

$$\sum_a (\mathbf{V}_a \cdot \nabla)\mathbf{E}_a = 0; \qquad \sum_a (\mathbf{V}_a \cdot \nabla)\mathbf{B}_a = 0. \tag{48}$$

In other words, every additional term in (31)-(32) (as well as in (41)-(42)) disappears and we obtain the original set of Maxwell's macroscopic equations (1)-(4) for continuous and closed (or going off to infinity) conduction currents as valid approximation.

To conclude this section we want to note that the set of eqs. (41)-(42) can be called as modified Maxwell-Hertz equations extended to one charge system. It is easy to see that in this form they are completely equivalent to modified Maxwell-Lorentz equations (31)-(32) obtained with the use of the balance equation. Thus, differential and integral approaches to extend the original Maxwell's equations seems to give the same result.

V. RELATIVISTICALLY INVARIANT FORMULATION OF ALTERNATIVE FIELD EQUATIONS

Let us write once again the alternative form of Maxwell-Lorentz equations explicitly for a single moving particle that is a source of electric and magnetic fields simultaneously:

$$div\,\mathbf{E} = 4\pi\varrho \tag{49}$$

$$div\,\mathbf{B} = 0 \tag{50}$$

$$curl\,\mathbf{B} = \frac{4\pi}{c}\varrho\mathbf{V} + \frac{1}{c}\left\{\frac{\partial\mathbf{E}}{\partial t} - (\mathbf{V}\cdot\nabla)\mathbf{E}\right\} \tag{51}$$

$$curl\,\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t} + \frac{1}{c}(\mathbf{V}\cdot\nabla)\mathbf{B} \tag{52}$$

at the same time with the balance equation:

$$\frac{d\varrho}{dt} + div\,\varrho\mathbf{V} = 0. \tag{53}$$

In the second section we already mentioned the necessity of a constant taking into account respective additional conditions for each part of total time derivative in (45)-(46). In other words, it corresponds to the separation within the total values into two independent components. An explicit use of this conditions in the basic field equations (51)-(52) can be represented as follows:

$$curl\,\mathbf{B} = \frac{4\pi}{c}\varrho\mathbf{V} + \frac{1}{c}\left\{\frac{\partial\mathbf{E}^*}{\partial t} - (\mathbf{V}\cdot\nabla)\mathbf{E}_0\right\} \tag{54}$$

$$curl\,\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}^*}{\partial t} + \frac{1}{c}(\mathbf{V}\cdot\nabla)\mathbf{B}_0 \tag{55}$$

where the total field values are compound by two independent parts:

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}^* = \mathbf{E}_0(\mathbf{r} - \mathbf{r}_q(t)) + \mathbf{E}^*(\mathbf{r}, t) \tag{56}$$

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}^* = \mathbf{B}_0(\mathbf{r} - \mathbf{r}_q(t)) + \mathbf{B}^*(\mathbf{r}, t) \tag{57}$$

Here we note that quasistatic field components $\mathbf{E}_0$ and $\mathbf{B}_0$ (further, we shall distinguish them by putting subindex "0") depend only on the point observation and on the source position at an instant whereas time varying-fields $\mathbf{E}^*$ and $\mathbf{B}^*$ depend explicitly on time in a fixed point. The separation procedure may by similarly extended to the electric and magnetic potentials introduced as:

$$\mathbf{E} = -grad\,\varphi; \qquad \mathbf{B} = curl\,\mathbf{A} \tag{58}$$

where

$$\varphi = \varphi_0 + \varphi^* \qquad \text{and} \qquad \mathbf{A} = \mathbf{A}_0 + \mathbf{A}^*. \tag{59}$$

The invariance of the basic equations of the classical electrodynamics under Lorentz transformations demands the system to be covariant in form. Although this requirement of covariance is satisfied by the conventional representation of Maxwell-Lorentz equations (6)-(9), there are some difficulties with application of Lorentz gauge condition to quasistatic fields. Before discussing it, let us prove the invariance of alternative Maxwell-Lorentz (or Maxwell-Hertz) equations in form of (49)-(52). This system can be further simplified by introducing from (55) a so-called electromotive force:

$$\mathbf{E} = -grad\,\varphi - \frac{1}{c}\frac{\partial \mathbf{A}^*}{\partial t} - \frac{1}{c}(\mathbf{V} \times \mathbf{B}_0). \tag{60}$$

Substituting $\mathbf{E}$ in (54) we must use separately quasistatic and time-varying parts of total electromotive force (60) in the following way:

$$\mathbf{E}_0 = -grad\,\varphi_0 - \frac{1}{c}(\mathbf{V} \times \mathbf{B}_0) \qquad \text{and} \qquad \mathbf{E}^* = -grad\,\varphi^* - \frac{1}{c}\frac{\partial \mathbf{A}^*}{\partial t}. \tag{61}$$

The eqs. (50) and (52) (or (55)) are satisfied automatically and we are left with the two differential equations:

$$\Delta \mathbf{A} = -\frac{4\pi}{c}\varrho \mathbf{V} + \mathbf{F} \tag{62}$$

$$\Delta \varphi = -4\pi\varrho + \mathcal{F} \tag{63}$$

where

$$\mathbf{F} = grad\,div(\mathbf{A}_0 + \mathbf{A}^*) - \frac{1}{c}(\mathbf{V}\cdot\nabla)grad\,\varphi_0 + \frac{1}{c}\frac{\partial}{\partial t}(grad\,\varphi^*) - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}^*}{\partial t^2} \tag{64}$$

$$\mathcal{F} = -\frac{1}{c}div\left(\frac{\partial \mathbf{A}^*}{\partial t}\right). \tag{65}$$

The second term in (64) can be easily transformed using mathematical operations of field theory:

$$(\mathbf{V}\cdot\nabla)grad\,\varphi_0 = grad(\mathbf{V}\cdot grad\,\varphi_0) - (\mathbf{V} \times curl\,grad\,\varphi_0). \tag{66}$$

Since *curl grad* is always equal to zero, we can rewrite $\mathbf{F}$ in new form:

$$\mathbf{F} = grad\left\{div\,\mathbf{A}_0 - \frac{1}{c}(\mathbf{V}\cdot grad\,\varphi_0) + div\,\mathbf{A}^* + \frac{1}{c}\frac{\partial \varphi^*}{\partial t}\right\} + \frac{1}{c^2}\frac{\partial^2 \mathbf{A}^*}{\partial t^2}. \tag{67}$$

The principal feature of (67) consists in the fact that all quasistatic and time-varying components of total electric and magnetic potentials enter independently and, therefore can be characterized by respective gauge conditions:

$$div\,\mathbf{A}_0 - \frac{1}{c}(\mathbf{V}\cdot grad\,\varphi_0) = 0 \tag{68}$$

$$div\,\mathbf{A}^* + \frac{1}{c}\frac{\partial\varphi^*}{\partial t} = 0. \tag{69}$$

As a result, quasistatic fields turn out to be related through the novel gauge (68) which is relativistically invariant and contains a well-known relationship between the components of electric and magnetic field potentials of uniformly moving charge [15]:

$$\mathbf{A}_0 = \frac{\mathbf{V}}{c}\varphi_0. \tag{70}$$

Recall that in the common point of view, electric and magnetic potentials of uniformly moving charge are implicit time-dependent functions. In this respect, (68) can be regarded as an extension of (70) to all quasistatic quantities of electromagnetic field. On the other hand, a unique reliable way to obtain (70) was based on the use of Lorentz transformation. Here, it should be specially stressed the essential role of the so-called *convection displacement current* conception in deducing (70). So far, only time-varying fields were interrelated explicitly due to Maxwell's displacement current. If one considers any stationary process, such connection between quasistatic fields was yet impossible since all displacement currents vanish from the Maxwell-Lorentz equations. Contrary to this, the fundamental symmetry between quasistatic electric and magnetic fields is now based on the equivalent conception of the *convection displacement current*.

Another important aspect of the present approach can be attributed to the verification of the limited character of the Lorentz gauge that now is applicable only to the time-varying field components. In fact, there are some difficulties in the conventional electrodynamics concerning the inconsistency of this gauge with quasistatic potentials. Actually, in the framework of the traditional approach, the Lorentz gauge condition

$$div\,\mathbf{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0 \tag{71}$$

is assumed to be valid for total electric and magnetic potentials and is considered sufficient to hold the Maxwell's equations invariant under Lorentz transformation. However, in the quasistationary approximation, the Lorentz condition in every frame of reference takes the form of so-called radiation gauge [21]:

$$div\,\mathbf{A} = 0. \tag{72}$$

On the other hand, due to the relation (70) between electric and magnetic quasistatic potentials (72) is not satisfied directly. To make (72) consistent with (70) in the given frame, it is suitable to put an additional condition on the electric potential so that we arrive to the so-called Coulomb gauge:

$$div\,\mathbf{A} = 0 \qquad \text{and} \qquad \varphi = 0. \tag{73}$$

In mathematical language the invariance of quasistatic fields involves stronger limitations than those imposed previously by Lorentz gauge. Generally speaking, the conventional classical electrodynamics must admit more than one invariance principle since every time we make a Lorentz transformation, we need also simultaneously transform all physical quantities in accordance with the Coulomb gauge (73). This problem was widely discussed and in the language adopted in the general Lorentz group theory, is known as *gauge dependent representation (or joint representation)* of the Lorentz group [21]. In fact, it means an additional non-relativistic adjustment of electric potential, every time we change the frame of reference. This difficulty takes no place if we introduce the entirely relativistic gauge (68) for quasistationary potentials.

A rigorous consideration of (62)-(65) gives another important conclusion: simultaneous application of two independent gauge transformations (68)-(69) decomposes the initial set (49)-(52) into two uncoupled pairs of differential equations, namely:

$$\Delta \mathbf{A}_0 = -\frac{4\pi}{c}\varrho \mathbf{V} \tag{74}$$

$$\Delta \varphi_0 = -4\pi\varrho \tag{75}$$

at the same time with the homogeneous wave equations:

$$\Delta \mathbf{A}^* - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}^*}{\partial t^2} = 0 \tag{76}$$

$$\Delta \varphi^* - \frac{1}{c^2}\frac{\partial^2 \varphi^*}{\partial t^2} = 0. \tag{77}$$

Likewise (70), Poisson's second order differential equations (74)-(75) for electric and magnetic potentials are in agreement with the conventional approach in the steady state and can be considered as valid extension to all quasistatic potentials.

The general solution, as one would expect, satisfies a pair of uncoupled inhomogeneous D'Alembert's equations that can be verified by summing up (74)-(75) and (76)-(77) (here we omit premeditatedly all boundary conditions for the sake of simplicity):

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\varrho\mathbf{V} \tag{78}$$

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi\varrho \tag{79}$$

where the total values $\mathbf{A}$ and φ are defined by (59).

The same result has been obtained in [12] independently from the analysis of value boundary conditions for inhomogeneous D'Alembert's equations. As a matter of fact, there has been shown that mathematically complete general solution of Maxwell's equations must be written as a linear combination of two non-reducible functions with implicit and explicit time-dependence. Additionally, the present approach demonstrates the invariance of (78)-(79) and therefore (49)-(52), if and only if two gauge conditions (68)-(69) are satisfied by respective components of the total field values.

To conclude this section, some remarks must be made concerning the empirical and axiomatic status of Lorentz force conception in the electron theory formulated by Lorentz. As we mentioned in *Introduction*, for the first time an explicit formula for mechanical force acting on a moving charge had been independently obtained in theoretical investigation by Thompson and Heaviside. More over, in the first version of Maxwell's theory published by the name "*On Physical Lines of Force*" (1861-1862) there was already admitted an unified character of a full electromotive force in the conductor at motion by describing it as [22,4]:

$$\mathbf{E} = -\nabla\varphi - \frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} + \frac{1}{c}(\mathbf{V}\times\mathbf{B}) \tag{80}$$

where the first term is the electrostatic force, the second one is the force of magnetic induction and the third one is the force of electromagnetic induction due to the conductor motion. It may be remarked here that later investigators began to distinguish between the electric force in a moving body and the electric force in the ether through which the body is moving and as a result, did not consider $\frac{1}{c}(\mathbf{V}\times\mathbf{B})$ as a full-value part of electric field that afterwards was argued by Hertz. This distinction had been profound by Lorentz in his electron theory and was tightly related to the special status of the Lorentz force conception. It also can be noted in the way how it forms part the formalism of the conventional field theory. Really, the mathematical form of (10) should be contrasted from the form of partial differential equations (6)-(9) that make them weakly compatibles with (10) contrary to that one would expect for the mathematical structure of a complete and consistent system.

There are another formal arguments against the axiomatic and empirical status of the Lorentz force. In classical mechanics one can find the very similar notion of Coriolis force. It is also depends on velocity, not position, and is not derivable from a potential. At the same time, being always normal to $\mathbf{V}$ it does not work and does not change the kinetic energy of a particle. In spite of this sameness, an explicit expression for the Coriolis force is deduced mathematically using the formalism of classical mechanics. On the other hand, there is a simple way of deriving of the Lorentz force, based on the transformation of the electromagnetic field vectors and the components of the Minkovski force. Thus, the expression for the Lorentz force can be obtained in a purely mathematical way from the general relations of the relativity theory (see, for instance, [17]). All this remarks make the status of the Lorentz force quite uncertain and pose the possibility of its reconsideration. In this respect, the present approach allows to treat a full electromotive force as completely unified conception that resembles the formalism adopted in early stages of electromagnetic field theory. In fact, the formula (60) used for the definition of full electromotive force includes automatically a term responsible for the motion. A negative sign of $-\frac{1}{c}(\mathbf{V}\times\mathbf{B}_0)$ can be easily understood. Contrary to (80) which corresponds to a moving medium and fixed external magnetic field, in our particular case, a point of observation (a site of a test charge) is fixed whereas a magnetic field is "moving" with $\mathbf{V}$. In the reference system where magnetic field is at "rest", the sign of the velocity is to be changed on the opposite and we come in agreement with the direction of the Lorentz force. Therefore, we can assume that for a consistent form of field equations there is no necessity to supplement them by the equation of motion.

VI. CONCLUSION

The above remarks motivate an important extension of the Maxwell's concept of displacement current to all stationary electromagnetic phenomena so that the fundamental symmetry between electric and magnetic fields (including all quasistatic fields) can be understood now as a consequence of the more general notion of displacement current. There are other compelling reasons for seeking an alternative form of Maxwell-Lorentz (as well as Maxwell-Hertz) equations that would contain such an extension. This approach has demonstrated some advantages over the conventional field description in eliminating a number of internal inconsistencies from Lorentz's electrodynamics. One of them is concerning the empirical and axiomatic status widely adopted in respect to the Lorentz force conception that can be modified by assuming an unified character of a total electromotive force. Thus, there is no more necessity to supplement the set of Maxwell's equations by the equation of motion since it is taken into account automatically.

The rigorous solution of fields equations shows the existence of two independent parts of field components correspond to longitudinal and transverse modes. More fundamentally, the independence of two parts of general solution must be attached on one hand to the implicit time dependence of longitudinal solutions and, on the other, to the transverse nature of explicitly time-varying fields. It arguments and corrects the conventional point of view about the transverse character of the total electromagnetic field and leads to the reformulation of the traditional field concept in terms of the so-called *electrodynamics dualism concept: simultaneous coexistence of instantaneous long-range (longitudinal) and Faraday-Maxwell short-rang (transverse) interactions*. More coherent and comprehensive analysis of this formulation and the compatibility of instantaneous action at a distance concept with the framework of relativistic classical electrodynamics can be found in [12]. In this work we prefer to confine our consideration by alternative form of Maxwell-Lorentz and Maxwell-Hertz equations.

Another aspect of the approach developed in this work relates to the deeper question of field gauge transformations. The existence of two independent types of fields (longitudinal and transverse) arrows us to determine independent gauge conditions for each other. The best way to end this article following A.O. Barut: "... *Electrodynamics and the classical theory of fields remain very much alive and continue to be the source of inspiration for much of the modern research work in new physical theories*" [21].

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